TECHNOLOGY

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

Page 1 of 46

http://www.measurementsgroup.com

A Measurements Group StrainSmart Hypertext Publication

TECHNOLOGY

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

Strain-Gage-Based Transducers

The resistance strain gage is, in itself, a transducer, since it converts mechanical deformation, or strain, into a corresponding electrical signal. Beginning, however, with the initial motivation for its invention, the strain gage has always been applied not only to the measurement of strain for stress analysis purposes, but also as the sensing element in numerous types of transducers for measuring other mechanical variables such as force, torque, and pressure. This article deals with transducers in the latter sense, and is intended to provide guidance for designing and building a variety of simple transducers.

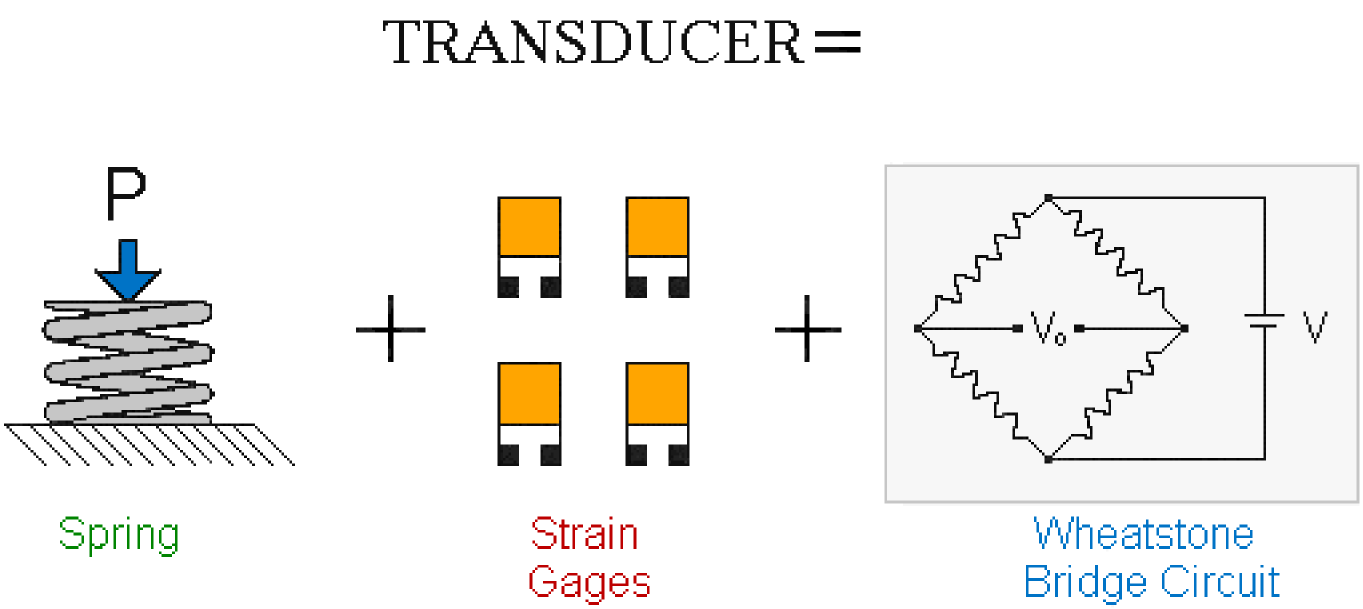
As pointed out by Robert Hooke in the 17th century, the deformation of a metal spring is directly proportional to the applied load. Although Hooke's measurements were of limited accuracy due to the instruments available in his time, it is a fact, nevertheless, that common structural metals such as steel and aluminum alloys are essentially linear in their elastic stress/strain characteristics. The foregoing applies, of course, only at stress levels below what is now referred to as the "proportional limit". A further restriction is that the overall deformation of the spring under the applied load is small enough that the spring geometry does not change sensibly. Otherwise, neither the stress nor the strain will be proportional to the load.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

All strain-gage-based transducers derive from Hooke's law of proportionality between stress and strain. Instead of measuring spring displacement as Hooke did, the unit deformation (strain) in the spring is converted into an electrical signal by strategically placed strain gages mounted on the spring. Such transducers normally consist of three main components (shown below): a metal spring, an arrangement of several strain gages (usually two or four), and a Wheatstone bridge circuit.



Principal Components of a Strain-Gage-Based Transducer

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

The principal load-bearing member in a transducer is commonly referred to as the *spring element*. Although it is a spring in the broad sense of the word, it differs from conventional springs in one important characteristic. Ordinarily, the purpose of a spring in a mechanical system is to store and release energy in a manner analogous to the function of a capacitor in an electrical circuit. Since the stored energy for a given applied force is proportional to the square of the displacement, such springs are usually designed for relatively high compliance. The spring element of a transducer, however, should normally be designed for extremely low compliance and minimum stored energy (except in the case of displacement transducers, treated briefly in a later section). Transducer spring elements may take many different physical forms, depending largely on the mechanical variable to be measured; but they seldom look like conventional springs.(Ref. 1)

Of primary importance to the functioning of a transducer are the strain gages, which must accurately and repeatably sense the strains in the spring as a measure of the applied load. For satisfactory transducer performance, careful consideration must be given to the location and orientation of the gages on the spring element relative to the known strain distribution in terms of magnitude, sign, and direction. Proper installation and wiring are also critical if accurate, stable gage operation is to be achieved.

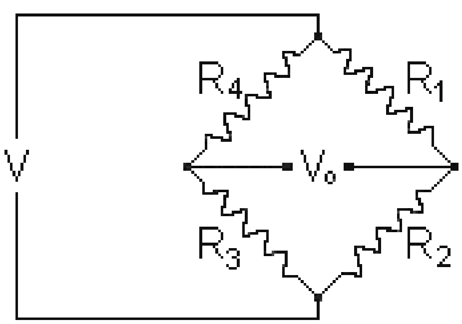
(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

The Wheatstone bridge circuit is a prominent component in determining transducer performance. Aside from its convenience in detecting small resistance changes, the bridge circuit provides another, and much more significant feature which is widely exploited in transducer design. The reference here is to the unique capability of the bridge circuit for performing algebraic addition or subtraction of resistance changes according to whether they occur in opposite or adjacent arms of the bridge (shown below).

WHEATSTONE BRIDGE ARITHMETIC



For the small resistance changes typically employed in strain-gage-based transducers, the bridge circuit output (V_o / V) is proportional to the *algebraic sum* of the changes in opposite arms (1 and 3, and 2 and 4), and to the *algebraic difference* of those in adjacent arms (1 and 2, 2 and 3, 3 and 4, and 4 and 1). Thus for example, if resistance changes of equal magnitude and the same sign occur in arms 1 and 3, so that $\Delta R_3 = \Delta R_1$, the resulting output is proportional to $\Delta R_1 + \Delta R_3 = \Delta R_1 + \Delta R_1 = 2 \times \Delta R_1$. On the other hand, if these resistance changes are numerically equal but opposite in sign, so that $\Delta R_3 = -\Delta R_1$, there is no output since $\Delta R_1 + \Delta R_3 = \Delta R_1 + (-\Delta R_1) = 0$. The same arithmetical operation takes place for resistance changes in arms 2 and 4.

In contrast, resistance changes in adjacent bridge arms are subtracted algebraically. Therefore, if $\Delta R_2 = \Delta R_1$ (in magnitude and sign), the bridge output due to these resistance changes is zero because $\Delta R_1 - \Delta R_2 = \Delta R_1 - \Delta R_1 = 0$. But when the same resistance changes are opposite in sign, with $\Delta R_2 = -\Delta R_1$, the bridge output is proportional to $\Delta R_1 - \Delta R_2 = \Delta R_1 - (-\Delta R_1) = 2 \times \Delta R_1$. Such algebraic subtraction occurs in all four pairs of adjacent bridge arms.

As demonstrated by the examples here in, Wheatstone bridge arithmetic is widely used in transducers to provide augmentation of desired signal components and cancellation of undesired components. Such signal transformations are accomplished by judicious placement of strain gages on the spring element (relative to its strain distribution) combined with assignment of the gages to the appropriate arms of the bridge circuit for algebraic addition or subtraction.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

Although transducer applications of the strain gage date from its invention in the late 1930's, the last twenty years have seen a tremendous growth in this area. Most of the growth has occurred in what are called "O-E-M" transducers; i.e., those intended for incorporation as a component in some other product. A good example of this is the currently widespread use of strain-gage-based transducers in the produce scales of most supermarkets. There are also, however, many manufacturers who supply off-the-shelf transducers, available in numerous types and load ratings, for measuring common mechanical variables such as force, torque, and pressure.

As a rule, when the need for a transducer arises, the best solution to the problem, if feasible, is to purchase a transducer from an established manufacturer. Commercial transducers can be obtained with very close specifications on accuracy, linearity, and other performance criteria. Moreover, the commercial transducer is apt to be less expensive than one of the homemade variety when all of the costs incurred in designing and building a transducer are accounted for.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

There are occasions, however, when a Do-It-Yourself (D-I-Y) transducer is the most appropriate or only practicable solution to a measurement problem. In some cases, for instance, there may be no means for introducing a commercial transducer into the load path. It then becomes necessary to make some member of the load train into a transducer by instrumenting it with strain gages, and calibrating it for its strain-versus-load characteristics. Similarly, it may be found that no standard commercial transducer will fit into the available space or adapt to the mechanical connections where a load measurement must be made. In short, a D-I-Y transducer is appropriate when warranted by task-- and/or situation --specific factors such as special configuration requirements, convenience, timing, capital equipment budget, etc.

It should be realized, in general, that the resulting transducer, although satisfactory for its intended purpose, may not compare well in performance to a high-quality commercial unit. This is to be expected since commercial transducer manufacturers, with years of experience and special manufacturing facilities, are normally in a much better position to optimize transducer performance than the ordinary laboratory or engineering department which sets out to make a single transducer.

TECHNOLOGY

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

Building a D-I-Y Transducer

The first step in achieving a suitable D-I-Y transducer is to arrive at the spring element material and design. With respect to the material, there are many alloys which would suffice for a moderately accurate transducer, but the performance of the transducer can be improved by the exercise of some judgement in material selection. It is important, of course, that the material have highly linear stress/strain characteristics. Other desirable material properties are low hysteresis and minimal creep under sustained load. Machinability of the alloy is also a factor, particularly if the spring element configuration is at all complex. Among the steel alloys, 4140, 4340, and 17-4 PH stainless steel are good candidates, and are widely used by commercial transducer manufacturers since they can be machined in the unhardened condition and then heat treated to develop the desired properties. Although not ordinarily thought of as spring materials, aluminum alloys such as 2024-T8 are also suitable, and often selected for low-capacity transducers.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

Mechanical design details of the spring element naturally play an important role in transducer performance. As an example, the spring element should be designed for very low overall compliance; i.e., for minimal displacement at the point of load application under rated load. This serves to prevent sensible changes in the spring geometry which would otherwise introduce nonlinearity as the load increases. At the same time, the spring should incorporate an area for strain gage installation where the strain level at rated load is in the range of 1000 to 1500 $\mu\varepsilon$ to provide sufficient output signal for measurement with conventional instruments. In general, the section where the strain gages are mounted should be the most highly strained region in the spring element. An unduly thin section at the strain gage site should be avoided, since it can lead to creep of the spring and to undesirable thermal effects due to the poor dissipation of the self-generated heat from the strain gages. The apparent need for a thin section at the gage site tends to occur in low-capacity bending-beam transducers. This problem can often be overcome by selecting an aluminum alloy as the spring material, permitting a thicker section for the same rated load and design strain level.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

There is usually little difficulty in selecting appropriate strain gages for a D-I-Y transducer. Normally, for operating temperatures less than about 200°F (95°C), constantan gages with polyimide backing (Micro-Measurements A-alloy gages, from the EA- or CEA- series) should serve very well. The gage length can be selected for compatibility with the available mounting space, but gages of 1/8 in. (3mm) or larger gage length are preferred for ease of installation and best operating stability. It is also preferable to employ 350-ohm (instead of 120-ohm) gages, since the higher resistance will reduce the gage self-heating at any given level of excitation voltage.

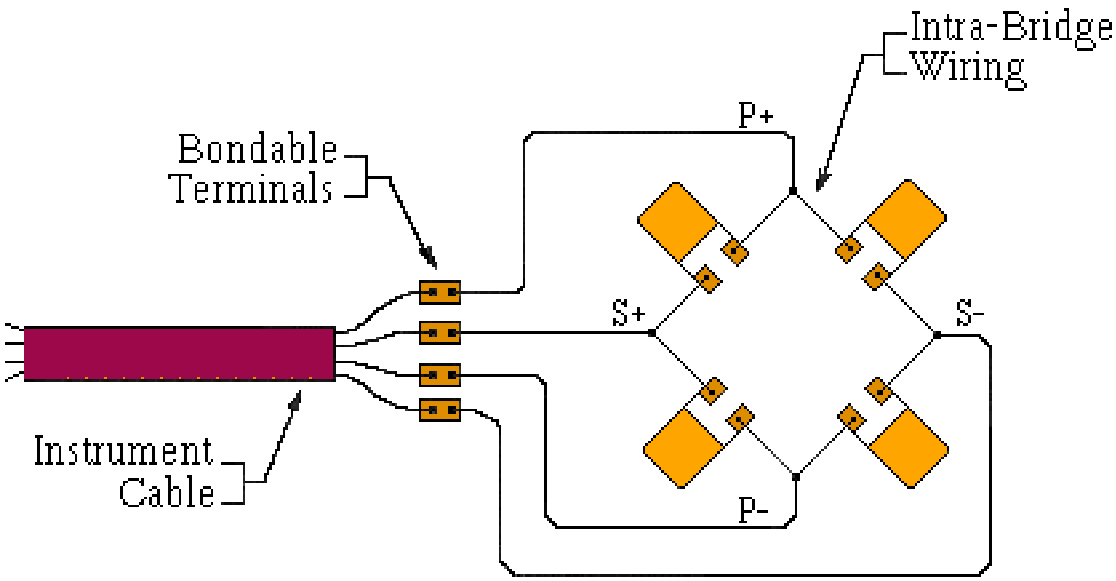
Special care in strain gage installation procedures will generally pay dividends in transducer performance. The gages should, for instance, be precisely located and oriented to sense only the intended strains as related to the applied load. Proper installation procedures for surface preparation, bonding, and wiring should also be performed by meticulously following the directions given in the relevant Micro-Measurements Instruction Bulletins. With the gage installation completed as instructed (including removal of all soldering flux), a protective coating should be applied over the gages.

For satisfactory transducer performance, it is usually necessary to have at least two gages mounted on the spring element; and, more often, four gages are employed. Multiple gages should always be from the same manufacturing lot to assure the greatest uniformity in gage properties. These gages are connected in different arms of the bridge circuit to take advantage of the circuit capability for the addition and subtraction of output signal components.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)



Typical Transducer Wiring Arrangement

The "intra-bridge" wiring, which connects the gages to form the bridge circuit shown above, can be done with short lengths of small-diameter copper wire (say, 34AWG, or 0.16mm). In order to preserve the resistive symmetry of the circuit, the same physical length of connecting wire should be used in each arm of the bridge. Four-conductor cable, wired to the bridge circuit through bondable terminals, is ideal for power supply and signal leads to the instrument (see Micro-Measurements Catalog A-110 for detailed information on gage installation and wiring supplies). The solder joints at wiring connections should always be small and neat, with smooth, shiny surfaces. When the wiring is completed, all wires should be adhered firmly to the surface of the spring element; usually with a suitable protective coating.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

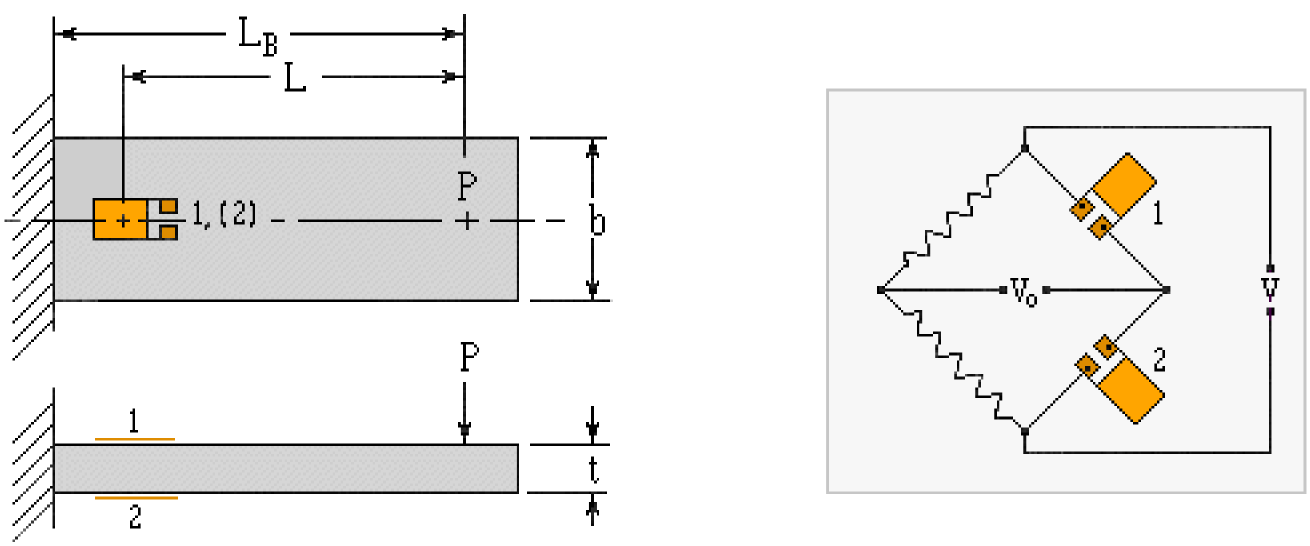
Calibration of the finished transducer is always necessary for accurate measurement of the intended mechanical variable. For the simple spring configurations described in the following sections, approximate equations are given relating the expected output signal from the bridge circuit to the mechanical input. These relationships are labeled as approximate for any of several reasons. To begin with, all of them necessarily involve the elastic modulus of the spring material (either directly or through the shear modulus), and sometimes the Poisson's ratio of the material as well. These properties are ordinarily characterized by uncertainties of from 1% to 3%. Additional uncertainties arise from the tolerance (typically ±0.5%) on the gage factor of the strain gages, and from dimensional tolerances on the spring element and loading geometry. In addition, there is always a small but finite signal attenuation due to the resistances of the intra-bridge wiring and the power supply leads. And, of course, for any but the simplest spring configurations, the mechanics relationship between load and strain is frequently an approximation in itself.

TECHNOLOGY

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

Bending Beam Transducers

Probably the most widely used transducer spring configuration is some form of a beam in bending. The cantilever beam is selected as an example here because of its basic simplicity and well known mechanical properties. A beam of this type can often be designed to provide a moderately accurate means for measuring forces or weights. The simplest form of cantilever beam for a D-I-Y transducer is shown below, along with the corresponding bridge circuit. The beam proportions should be determined (with respect to the rated load) to produce a suitable level of strain at the gage site, near the root of the beam, without excessive deflection at the point of load application. In this case, two strain gages are installed on the longitudinal centerline of the beam; with one on the top surface and one on the bottom, at the same distance from the root of the beam.



Cantilever Beam Force Transducer,with Half-Bridge Circuit

(continued...)

Page 13 of 46

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

When the beam is loaded as indicated on previous page, the upper gage is strained in tension, and the lower one is strained equally in compression. These gages are connected in adjacent arms of the bridge circuit to benefit from the resulting algebraic subtraction of signal components (shown previously). Thus, since the resistance changes in both gages due to thermal output have the same sign and are nominally equal in magnitude, their effects are canceled by subtraction in the circuit. Furthermore, the resistance changes caused by the bending strains are equal in magnitude but opposite in sign, producing a bridge output signal which is double that of a single gage.* This occurs because $\varepsilon_2 = -\varepsilon_1$, and

$$\varepsilon_1 - \varepsilon_2 = \varepsilon_1 - (-\varepsilon_1) = 2 \times \varepsilon_1 .$$

---

* It is assumed, for this and all other transducers described here, that the spring material is homogeneous and is isotropic in its elastic properties.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

Relevant design equations for the cantilever beam transducer shown previously are as follows:

$$\varepsilon_1 = \frac{6PL \times 10^6}{Ebt^2} \quad (1)$$

$$\varepsilon_2 = -\varepsilon_1 \quad (2)$$

$$\frac{V_o}{V} = \frac{F\varepsilon_1 \times 10^{-3}}{2} = \frac{3FPL \times 10^3}{Ebt^2} \quad (3)$$

$$D = \frac{4PL_B^3}{Ebt^3} \quad (4)$$

Where, for symbols not evident from the figure:

ε = strain, in $\mu\varepsilon$ units**
E = elastic modulus of the spring material
F = gage factor of the strain gages
V_o / V = bridge output, mV/V**
D = deflection at point of load application

It can be seen from Eq.(3) that, for a gage strain of 1500 $\mu\varepsilon$, and a gage factor of 2.0, the nominal bridge output will be 1.5mV/V.

**These designations are employed throughout this publication; that is:

ε = strain, in/in x 10^6 = m/m x 10^6
V_o/V = dimensionless output voltage in millivolts per volt of bridge supply voltage.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

The cantilever beam (shown previously) offers a number of advantages as a D-I-Y transducer. It is simple to design and fabricate, and it lends itself to easy strain gage installation. If the gages are accurately located and oriented on the beam centerline, it is also largely insensitive to extraneous loads. A load component along the beam axis, for example, should produce equal strains of the same sign in both gages, and the effects of these will be canceled by subtraction in the bridge circuit. The effects of gage strains caused by either side loading or torsion are also nullified, but for a different reason.

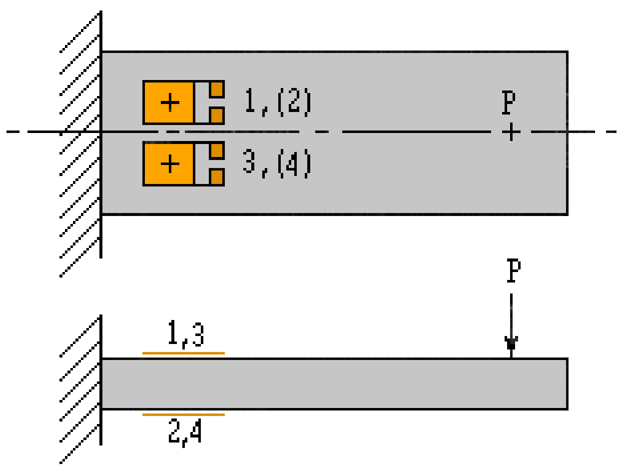
The total resistance change in a strain gage is the integral,or algebraic sum, of all such changes throughout the grid. Because the gages (shown previously) are mounted on the neutral axis with respect to bending caused by a side load, half of each gage is in tension, and half is in compression. Thus, the net resistance change in the gage due to a side load is zero. The strain distribution on the surface of the beam as a result of torsion is quite complex. It can be expected, however, that the strains on either side of the gage centerline will be equal and opposite in sign, and thus cancel. It is for this same reason, incidentally, that strain gages are insensitive to pure shear in the direction of, or perpendicular to, the grid lines.

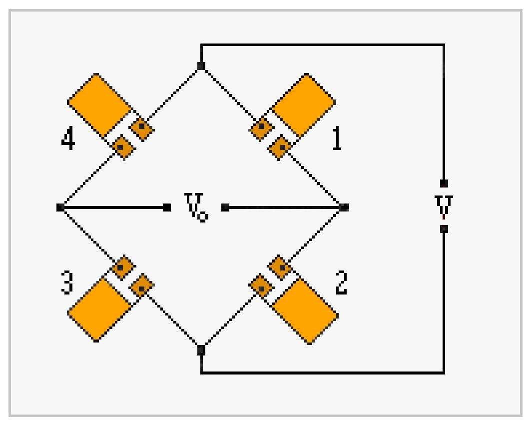
(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

The bridge output of a cantilever beam transducer can be doubled very easily by employing four strain gages instead of two. As shown below, pairs of gages are installed side-by-side on both the tensile and compressive surfaces of the beam. The gages are connected to form a complete ("full") Wheatstone bridge as indicated. With gages 1 and 3 on the tensile surface of the beam, and gages 2 and 4 on the compressive surface, this arrangement provides all of the compensating features of the preceding two-gage installation, and the bridge output is twice as great.





Cantilever Beam Force Transducer, with Full-Bridge Circuit for Doubled Output Signal
(Dimensions same as [previous figure](#))

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

Thus, since the bending strains produced by the load P are related as follows,

$$\varepsilon_2 = \varepsilon_4 = -\varepsilon_1 = -\varepsilon_3 \qquad (5)$$

The output expression, as a result of Wheatstone bridge arithmetic, becomes:

$$\frac{V_o}{V} = F\varepsilon \times 10^{-3} = \frac{6FPL \times 10^3}{Ebt_2} \qquad (6)$$

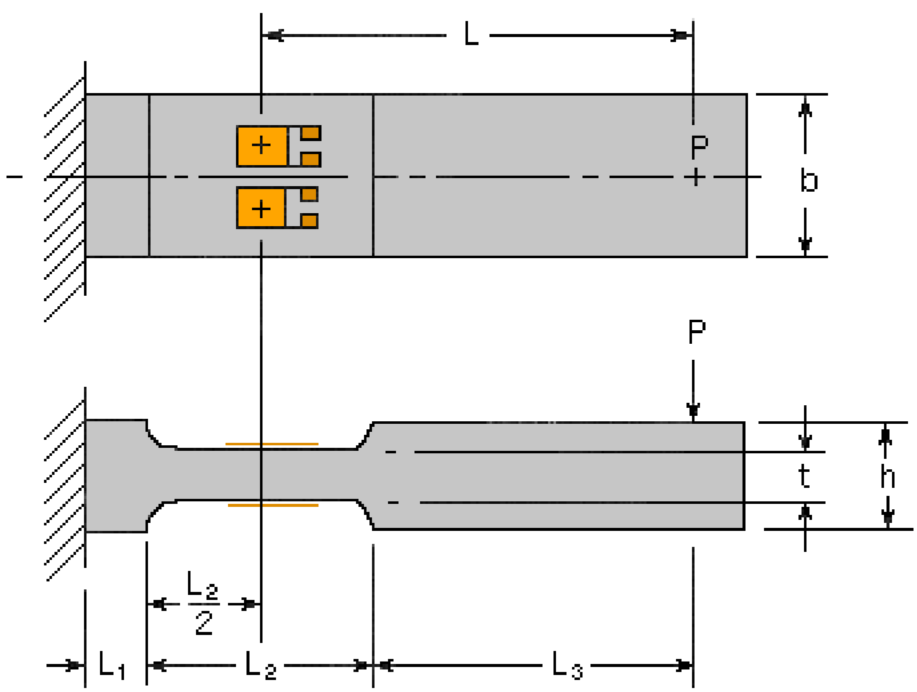
For a bending strain of 1500 $\mu\varepsilon$, and a gage factor of 2.0, Eq.(6) yields a bridge output of 3.0mV/V. The effects of extraneous loads, and of strain gage thermal output, are canceled within the bridge circuit. To achieve these characteristics it is necessary, of course, that all four gages be centered at the same section of the beam, and aligned parallel to the beam axis. In addition, gages 1 and 3 must be symmetrically located relative to the beam centerline, with gages 2 and 4 at corresponding locations on the opposing beam surface.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

A potential disadvantage of the cantilever beam spring element arises from its relatively high compliance as a load-bearing member. If there is measurable deflection of the beam at the point of load application, the bending moment at the gage site changes slightly with load, introducing a small nonlinearity in the output signal. An improvement in compliance can be made by considering that most of the beam length, although contributing to the deflection, serves only to provide the moment arm for generating the desired level of bending strain at the gage location. As a result, the compliance of the beam can be reduced noticeably by simply thickening the beam everywhere except in the strain gage area (shown below).



Mechanical Design of Cantilever Spring Element for Reduced Compliance. Deflection Under Load is a Function of b, t, h, L_1 *,*L_2 *,*L_3

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

Although this discussion is primarily restricted to static load considerations, it should be evident that the cantilever beam, because of its mass and compliance, tends to exhibit a low natural frequency. Another (and related) disadvantage of the cantilever beam configuration is that it can lead to impracticably bulky and massive spring elements for large rated loads.

Bending beam spring element configurations are not limited, of course, to cantilever beams. A centrally loaded, simply supported beam, for example, of the same overall beam dimensions, and at the same maximum strain level, is nominally four times as stiff as the cantilever. For still lower compliance, the beam can be rigidly built-in at both ends to prevent end rotation and induce two reversals of curvature along the beam length. For corresponding beam geometries and maximum strain levels, the built-in beam is about eight times as stiff as the cantilever. These and the many other beam configurations found in commercial transducers are not discussed further, since the design details necessary for satisfactory transducer operation are beyond the scope of this article.(Ref. 1)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

Column Load Cells and Tension Links

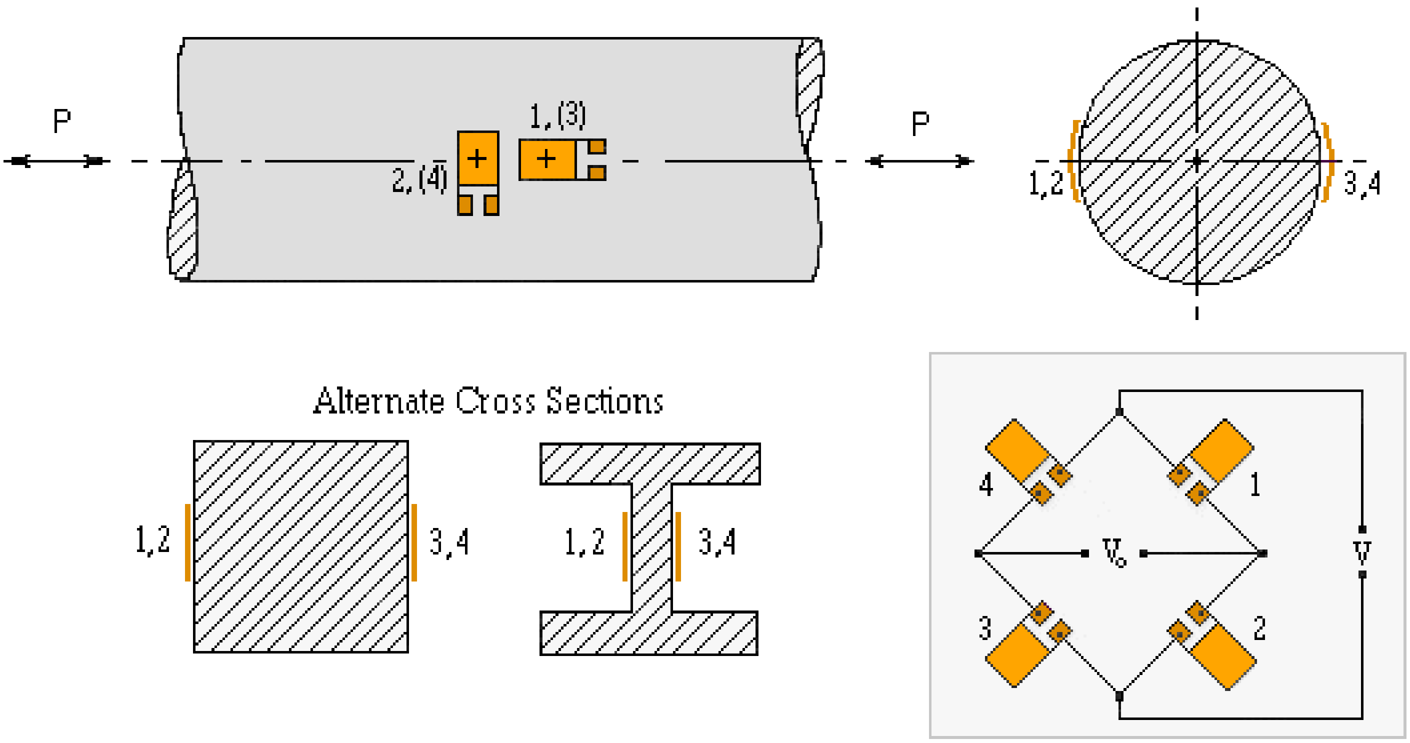
In some cases, the physical arrangement of the mechanical system where a load measurement must be made does not readily lend itself to the introduction of a bending beam transducer. In others, the required mechanical connections for load transfer to the transducer may dictate a different form of spring element. Such problems can sometimes be overcome with an axially loaded straight metal bar as the spring element. Because the stress in the bar is uniformly distributed over the cross section (except near the ends), and because the bar can usually be made quite short, this form of spring element is inherently much lower in compliance than a cantilever beam of similar size and design strain level. As a result of its low sensitivity in terms of strain versus load, the axially loaded spring design is best suited for the measurement of very large loads (in excess of, say, 10000 lb or 5000 kg).

A direct-stress spring element can be loaded, of course, either in compression as a column or in tension to form what is sometimes called a "tension link". Since the design considerations for the two cases are basically the same with respect to transducer characteristics, no distinction is made here. Obviously, the end fittings for transmitting the load and its reaction to the spring element will differ; and, for compression loading, the column length and cross section must be proportioned to avoid any instability from buckling.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)



Tension/Compression (Direct-Stress) Force Transducer

A representative direct-stress spring element is illustrated schematically (shown above). It can be noted that the physical arrangement of the strain gages on the spring, and their locations within the bridge circuit, are different from those for the bending beam. This is done to take advantage of the algebraic addition and subtraction characteristics of the bridge circuit relative to the strain distribution in the spring. For uniformly distributed uniaxial stress in the spring, there are no locations where the strains are equal in magnitude and opposite in sign. Thus, if the two axially oriented gages on the spring were connected in adjacent bridge arms (such as 1 and 2), their resistance changes would be subtracted, and there would be no bridge output under load. Instead, these two gages are connected in opposite bridge arms (1 and 3), causing their resistance changes to be added, and doubling the bridge output compared to a single gage. Moreover, if the transducer is subjected to an off-axis or transverse component of force, the resultant bending strains at the gage site, which should be the same in magnitude and opposite in sign, will be canceled due to algebraic addition of the resistance changes.

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

From the foregoing, it might appear that only two axially oriented gages on the spring will suffice for a satisfactory transducer. This arrangement cannot generally be recommended, however, because of thermal output effects. Since the thermal outputs of the gages are nominally equal in magnitude and have the same sign, the thermally induced resistance changes in the gages are also added, doubling the false output due to changing temperature. This situation is easily remedied by installing two more gages on the spring, but orienting the gages transversely to sense the Poisson strain. The transverse gages are connected in arms 2 and 4 of the bridge circuit. Because resistance changes in adjacent arms of the circuit are subtractive, the thermal output of gage 2 will cancel that of gage 1; and the same is true for gages 3 and 4. Thus, except for gage-to-gage variation in thermal output, which should be small if all gages are from the same manufacturing lot and are at the same temperature, there will be no bridge output with temperature changes. Furthermore, with the transverse gages connected in opposite bridge arms as shown previously, the effects of any bending strains in these gages are canceled, as they are for the axial gages.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

Following are the design relationships for a direct-stress transducer:

$$\varepsilon_1 = \frac{P \times 10^6}{EA} \tag{7}$$

$$\varepsilon_2 = \varepsilon_4 = -\nu\varepsilon_1 = -\nu\varepsilon_3 \tag{8}$$

$$\frac{V_o}{V} = \frac{F\varepsilon_1(1+\nu)\times 10^{-3}}{2 + F\varepsilon_1(1-\nu)\times 10^{-6}} \tag{9}$$

Or,

$$\frac{V_o}{V} \approx \frac{F\varepsilon_1(1+\nu)\times 10^{-3}}{2} = \frac{FP(1+\nu)\times 10^3}{2EA}$$
(9a)

$$D = \frac{PL_B}{EA}$$
(10)

Where,

A = cross-sectional area of the spring at the gage site
ν = Poisson's ratio of the spring material

In addition to canceling the strain gage thermal output, the transverse gages also serve to increase the output signal from the applied load. This occurs because the Poisson strain is opposite in sign to the longitudinal strain, and gages 2 and 4 are connected in adjacent bridge arms to gages 1 and 3. Since the Poisson's ratio for steel and aluminum alloys is about 0.3, incorporation of the two transverse gages does not double the bridge output, but only increases it by approximately 30 percent. This increase is reflected in the $(1 + \nu)$ term in the numerators of Eqs.(9) and (9a).

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

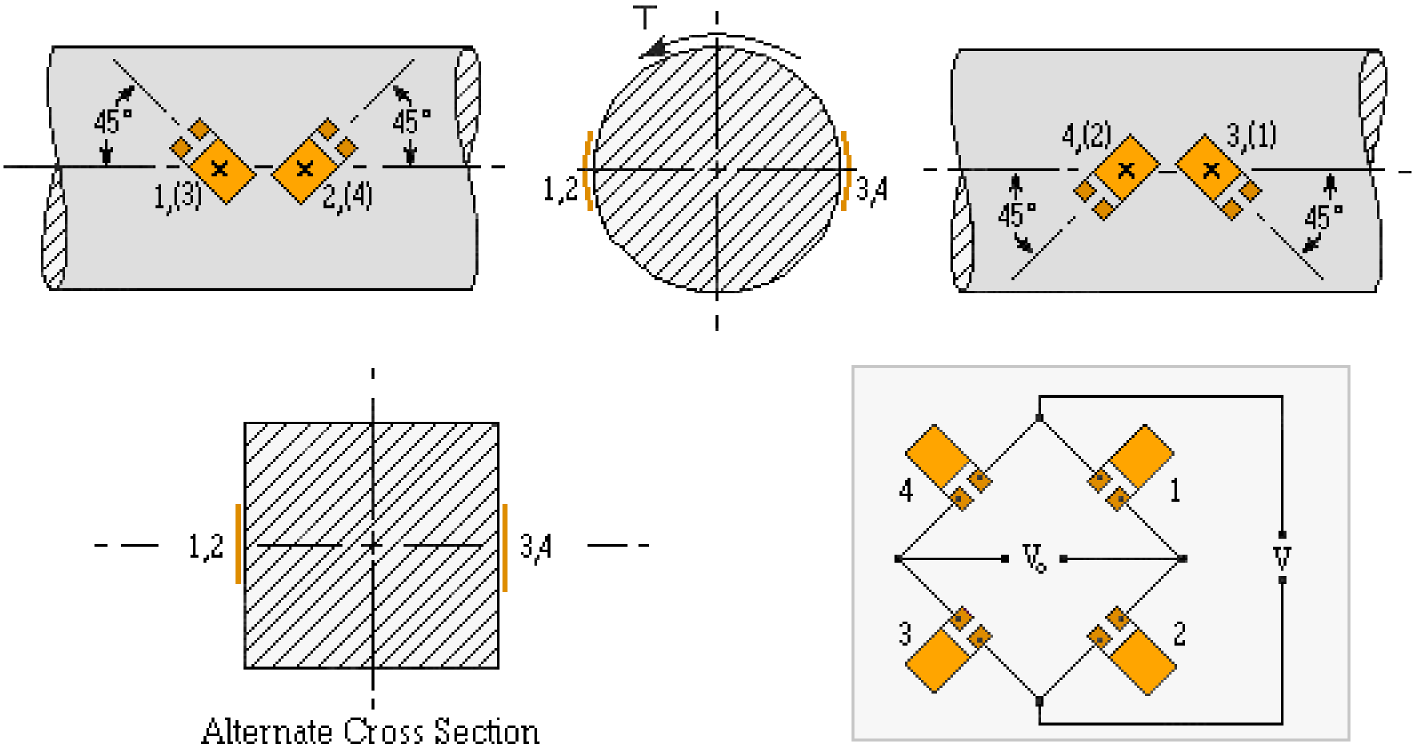
It can also be seen from the denominator of Eq.(9) that the bridge output is a nonlinear function of the axial strain in the spring. This effect is present because the transverse strains, although opposite in sign to the longitudinal strains, are not equal to them in magnitude. The nonlinearity is very small, however, and can ordinarily be ignored in the case of a D-I-Y transducer. If, for example, the gage factor of the gages is 2.0, and the axial strain is 1500 $\mu\varepsilon$, the nominal output of the transducer (ignoring the nonlinearity) is about 2mV/V. When the nonlinearity term in the denominator of Eq.(9) is considered, the difference in full-scale output is approximately 0.1 percent.

For simplicity of machining, it is often convenient to make the direct-stress spring element in the form of a circular cross-section bar. However, almost every aspect of strain gage installation, including gage location and orientation, can be performed more easily, and more precisely on a flat surface than on a curved one. Because of this, consideration can be given to using a bar of the appropriate square cross section, or using a larger diameter circular bar and grinding it back to a corresponding square section at the gage site. To achieve the bending cancellations described here it is necessary that the two axial gages on a round bar be located diametrically opposite one another, and at the same longitudinal position. The same is true for the two transverse gages. With a square bar, the pairs of like-oriented gages must also be at directly opposite positions on the bar, and their centerlines must coincide with the centerlines of the flat surfaces.

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

Torque Transducers

Although a less frequent requirement than force measurement, it is sometimes necessary to measure the torque in a mechanical system, and a strain-gage-based transducer is usually the preferred means for doing so. In the case of a circular cross-section shaft subjected to pure torque, there are equal shear stresses in both the longitudinal and circumferential directions, everywhere on the shaft surface. The shear strains corresponding to these stresses cannot be measured by longitudinally and circumferentially oriented strain gages, however, because a single strain gage is sensitive to only normal strains along its axis, and is totally insensitive to shear strains. But, from elementary mechanics of materials, it is known that pure shear produces plus and minus normal strains of equal magnitude at angles of plus and minus 45° from the shear direction. It is also known from mechanics principles that the difference between the two principal normal strains is equal to the maximum shear strain. Thus, the shear strains, which are proportional to the applied torque, can be measured with a pair of strain gages installed on the shaft surface and oriented at 45° on either side of a line parallel to the shaft axis. Most commonly, two pairs of such gages are mounted on the shaft, as shown below, to maximize the output signal and to provide for cancellation of extraneous signal components due to bending or direct stress.



Torque Transducer

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

Following are the design relationships for a circular cross section torque transducer:

$$\varepsilon_1 = \frac{T \times 10^6}{\pi G R^3}$$

(11)

$$\varepsilon_2 = \varepsilon_4 = -\varepsilon_1 = -\varepsilon_3$$

(12)

$$\frac{V_o}{V} = F\varepsilon_1 \times 10^{-3} = \frac{FT \times 10^3}{\pi G R^3}$$

(13)

Where: T = torque
G = E/(2 x (1 + *v*)) = shear modulus

For the direction of torque indicated previously, gages 1 and 3 are strained in tension, and gages 2 and 4 in compression; and the strains due solely to the applied torque are all equal in magnitude. When the gages are connected in the bridge circuit as shown, the resistance changes in every pair of adjacent bridge arms are opposite in sign, while those in opposite arms have the same sign. As a result, the bridge output, given by Eq.(13), is four times that of a single gage.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

On the other hand, the thermal outputs of all four gages are the same in sign and in nominal magnitude (if at the same temperature). Since the same resistance changes in all arms of the bridge have no effect on the state of bridge balance, the strain gage thermal output is canceled within the circuit. Such cancellation also occurs for strains due to a purely axial load on the shaft, and for the same reason. However, if the shaft is subjected to a bending moment in any plane, the resulting strains in diametrically opposed gages (1 and 3, and 2 and 4) are always numerically equal but opposite in sign. These are likewise canceled because the corresponding resistance changes appear in opposite pairs of bridge arms.

These applications of Wheatstone bridge arithmetic for augmenting the torsional output signal while canceling unwanted signal components require that the gages be precisely positioned and oriented. The centers of all four gages must lie on the same shaft section; they must be oriented along 45° helices on the shaft surface; and the centers of each pair of opposed gages must lie on a common diameter. The gage installation task can be simplified considerably by employing special “torque gages”, each of which contains two grids (on a common backing) precisely oriented at plus and minus 45° from the gage axis.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

Strain gage installation on a curved surface, particularly for the small radius of curvature typical of machine shafts, is never as easy as on a flat surface. Because of this, depending on the circumstances, it is sometimes preferable to design the shaft for a square section at the gage site. If the foregoing option is elected, it must be realized that the shear strain distribution over the flat surface is distinctly different from that on a circular shaft. The shear strain is maximum at the center of the flat face, and decreases more or less parabolically to zero at the edges. Because the shear strain magnitude falls off rather steeply from the center, the gages on each face should occupy no more than about 1/4 of the face width. There is no exact solution for the shear strain distribution across the face, but the maximum shear strain at the center is approximately 2.7 times that of a circumscribing circular section and about 0.95 that of an inscribing circular section.(Ref. 2)

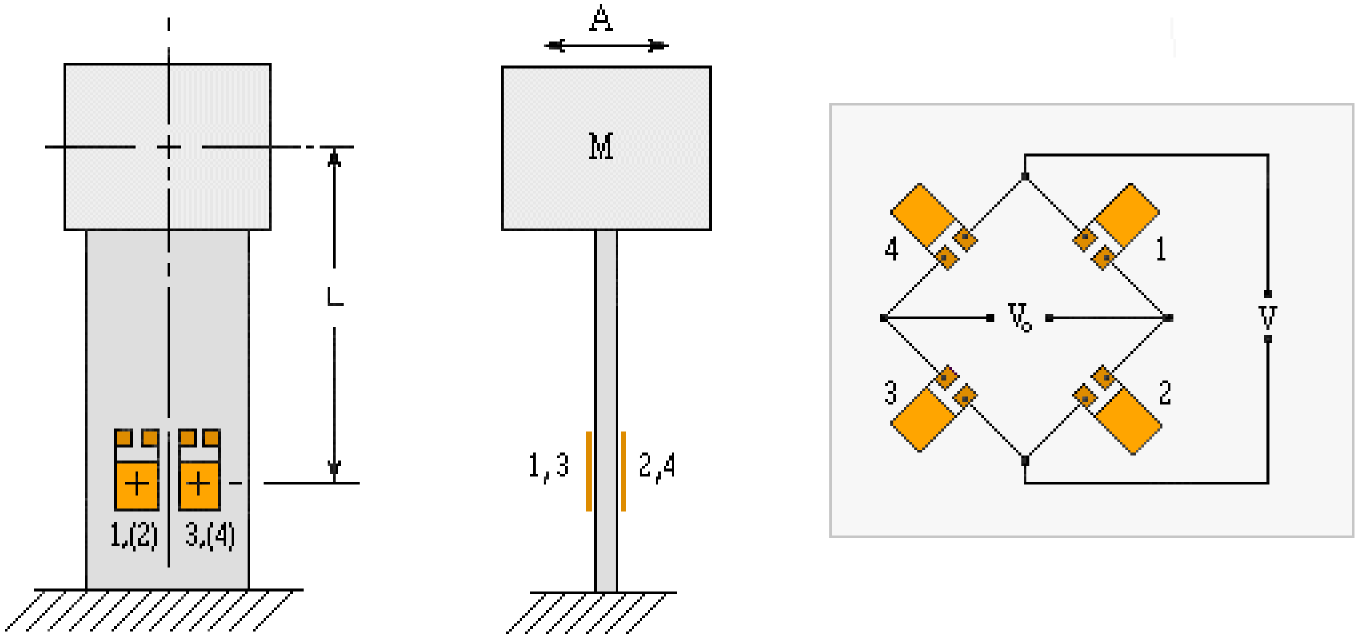
When the torque measurement is to be made on a continuously rotating shaft, the problem of getting the bridge output signal to stationary instrumentation naturally arises. Most commonly, slip rings are employed for this purpose, or telemetry with a rotating transmitter on the shaft. As an alternative, if either end of the drive train is accessible, it may be feasible to mount the driving or driven portion of the system in bearings (or flexure pivots) and measure the reaction torque with a conventional stationary transducer of the beam or column type.

TECHNOLOGY

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

Other Strain Gage Based Transducers

In general, any mechanical variable involving force or torque can be measured with a strain-gage-instrumented spring element. For example, a cantilever beam with a mass attached to the free end becomes an accelerometer when gages are installed and connected as shown below. The force applied to the beam is, by Newton's second law, equal to the product of the mass and acceleration. Thus, the circuit output can be calculated from Eq.(3), where the force P = M x A. To produce a satisfactory accelerometer it is necessary, of course, to consider practical aspects of the design such as damping and the system's natural frequency.



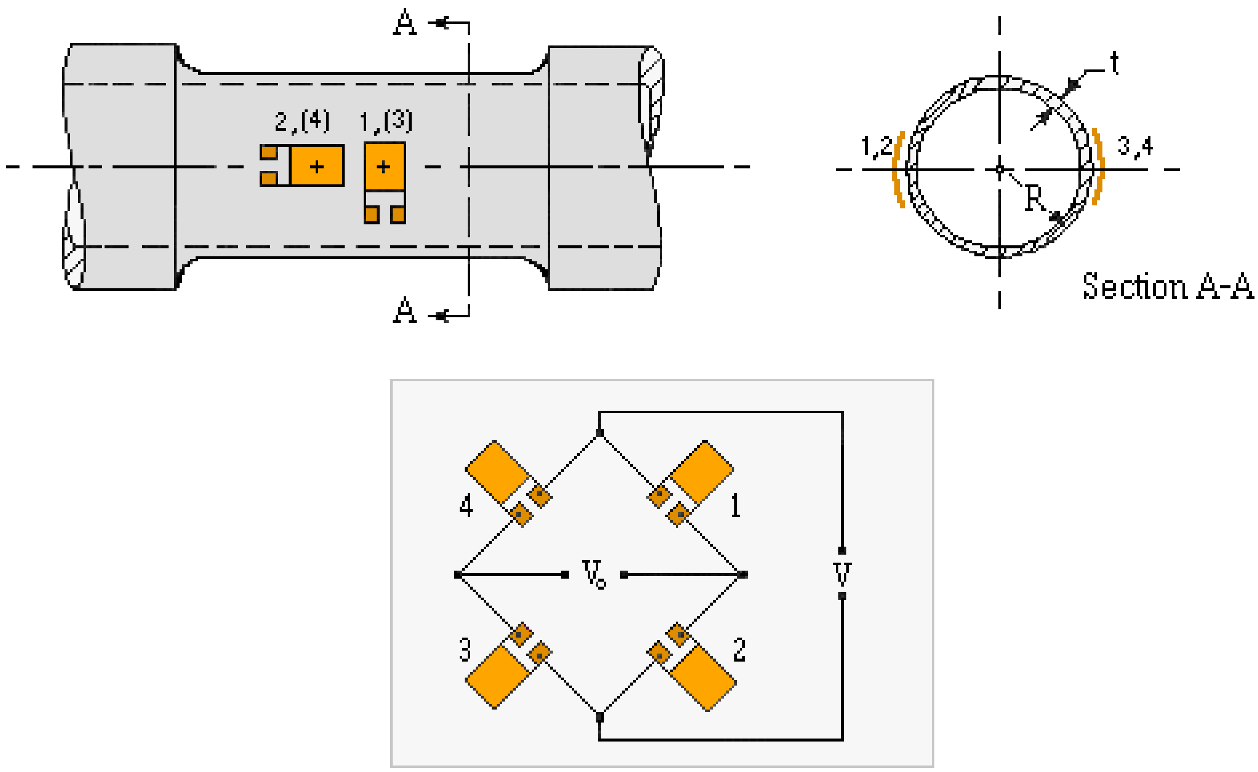
Concept For Strain-Gage-Based Accelerometer

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

Strain gages can also be used in the measurement of fluid pressure. One method of making a D-I-Y pressure transducer is to mount strain gages on a section of pipe (or tubing) where the wall of the pipe has been locally thinned to increase the strain level. Four strain gages can be installed on the pipe and connected in the bridge circuit as shown below. Gages 1 and 3 are aligned circumferentially to sense the hoop strain, while gages 2 and 4 sense the longitudinal strain. As for the previously described transducers, the thermal outputs of the gages will be nullified within the bridge circuit, since they are the same for all gages. The effects of bending strains in the pipe will also be canceled because the physically opposed gages (with opposite-sign bending strains) are connected in opposite bridge arms.



Pressure Transducer, Using A Thinned Pipe Wall.

(continued...)

Page 31 of 46

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

For a section of pipe which is unconstrained longitudinally, approximate design equations (based on thin-walled pressure vessel theory) are as follows:

$$\varepsilon_1 = \frac{PR}{Et}(1 - 0.5\nu) \times 10^6$$
(14)

$$\varepsilon_2 = \frac{PR}{Et}(0.5 - \nu) \times 10^6$$
(15)

$$\varepsilon_3 = \varepsilon_1 ; \varepsilon_4 = \varepsilon_2$$
(16)

$$\frac{V_o}{V} = \frac{F\varepsilon_1(1+\nu) \times 10^{-3}}{4 - 2\nu + 3\,F\varepsilon_1(1-\nu) \times 10^{-6}}$$
(17)

Where: P = pressure
R = mean radius of pipe
t = pipe wall thickness at gage site

It can be seen from Eq.(17) that the bridge circuit output for this case is a nonlinear function of the strain, and thus of the pressure. Assuming a gage factor of 2.0, a maximum hoop strain of 1500 $\mu\varepsilon$, and a Poisson's ratio of 0.3, the full-scale deviation from the linearity is about -0.2 percent. Ignoring the nonlinearity term in the denominator of Eq.(17),

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

$$\frac{V_o}{V} \approx \frac{F\varepsilon_1(1+\nu)\times 10^{-3}}{4-2\nu} \qquad (18)$$

Or, after substituting from Eq.(14),

$$\frac{V_o}{V} \approx \frac{F\left(\frac{PR}{Et}\right)(2+\nu-\nu^2)\times 10^{3}}{8-4\nu} \qquad (19)$$

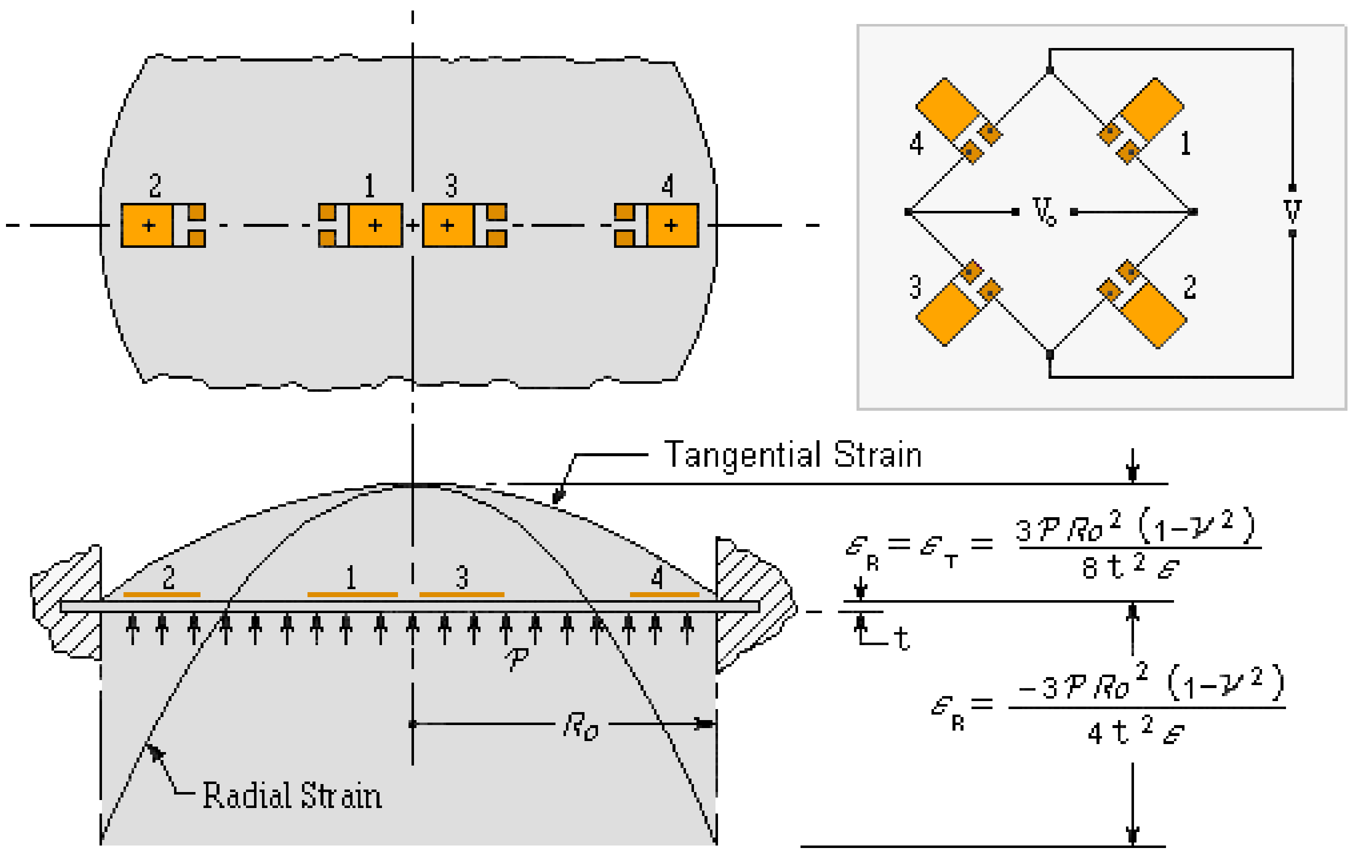
A more serious limitation (than nonlinearity) in this type of pressure transducer is the relatively small bridge output for a given maximum strain level. This occurs because the strains sensed by all four gages have the same sign. As a result, the longitudinal strains (which are equal to about 1/4 of the hoop strains) are effectively subtracted from the hoop strains due to the usual Wheatstone bridge arithmetic. With the preceding assumptions for gage factor and Poisson's ratio, the bridge output for a hoop strain of 1500 $\mu\varepsilon$ is only a little over 1.0mV/V. It should also be noted that the actual calibrated relationship between pressure and bridge circuit output may differ measurably from Eq.(19), depending on the mechanical constraints applied to the pipe by supports and connections.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

The most common form of strain-gage-based pressure transducer is a circular diaphragm, rigidly clamped completely around its edge (shown below). Commercially available transducers of this type are used very widely for industrial process instrumentation and control.



Diaphragm Pressure Transducer, With Strain Distribution And Gage Positioning

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

Because such a diaphragm exhibits strains of one sign in the region near its edge, and strains of the opposite sign in an area around the center, it lends itself to a full-bridge strain gage installation. This permits taking maximum advantage of the Wheatstone bridge characteristics for signal augmentation and cancellation of thermal output, as previously described for force and torque transducers. To avoid the necessity of installing four strain gages on a small diaphragm, the Micro-Measurements Division supplies integral strain gage assemblies containing four grids, with the grids appropriately configured for compatibility with the strain field of the diaphragm. The design criteria and output relationships for a diaphragm pressure transducer require more detailed treatment than can be given here, but these topics are covered in Measurements Group Tech Note TN-510, *Design Considerations For Diaphragm Pressure Transducers*.(Ref. 3)

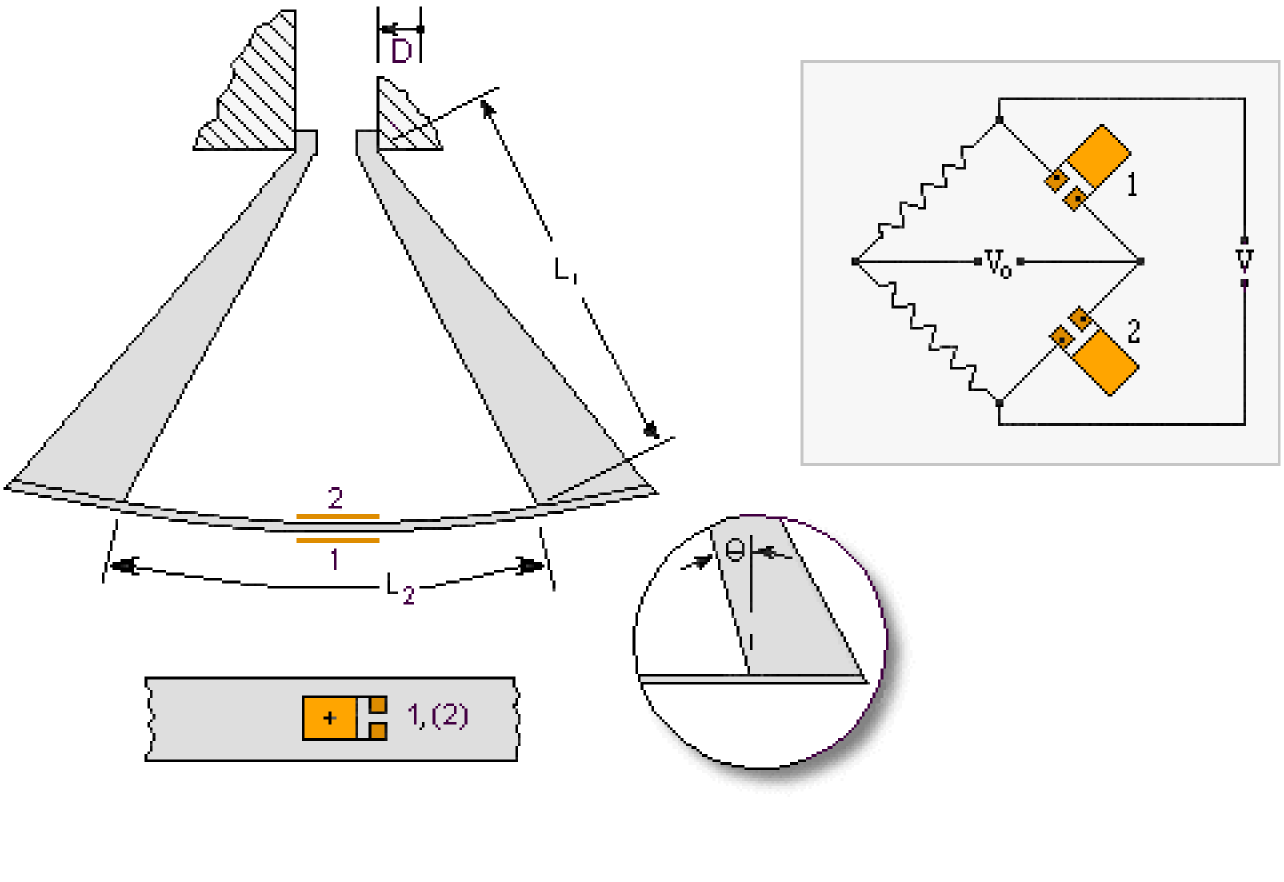
Beyond force- and torque-related variables, strain gages can also be used to measure small displacements. A strain-gage-based displacement transducer is usually made in the form of a lightweight spring having two or more strain gages on it to sense the spring flexure as its end points are displaced from one another. In contrast to the previously described transducers, the displacement transducer is normally a very high-compliance device to minimize the forces applied to the test object. Because these transducers are usually clipped to the test object in the manner of an extensometer, they are often referred to as "clip gages".

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

A representative clip gage configuration is illustrated schematically (shown below), where, in this case, two strain gages are installed on opposite sides of a thin section which is caused to bend as the distance between the attachment points changes. Since the strains sensed by the two gages are equal in magnitude and opposite in sign, the gages are connected in adjacent bridge arms to double the bridge output while canceling the thermal outputs of the gages.



Principle Of Clip Gage For Displacement Measurement

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

Approximate design relationships for the simple spring clip configuration in the figure shown on previous page are as follows:

$$\varepsilon_1 = \frac{tD\cos\theta}{2L_1L_2}\times 10^6$$
(20)

$$\varepsilon_2 = -\varepsilon_1$$
(21)

$$\frac{V_o}{V} = \frac{F\varepsilon_1 \times 10^{-3}}{2}$$
(22)

$$\frac{V_o}{V} = \frac{FtD\cos\theta\times 10^3}{4L_1L_2}$$
(23)

Although not evident from Eqs.(20)-(23), the relationship between end-point displacement and strain is actually nonlinear. Because of this, the equations are applicable with reasonable accuracy only for displacements which are small compared to the length L_1 of the attachment arms.

TECHNOLOGY

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

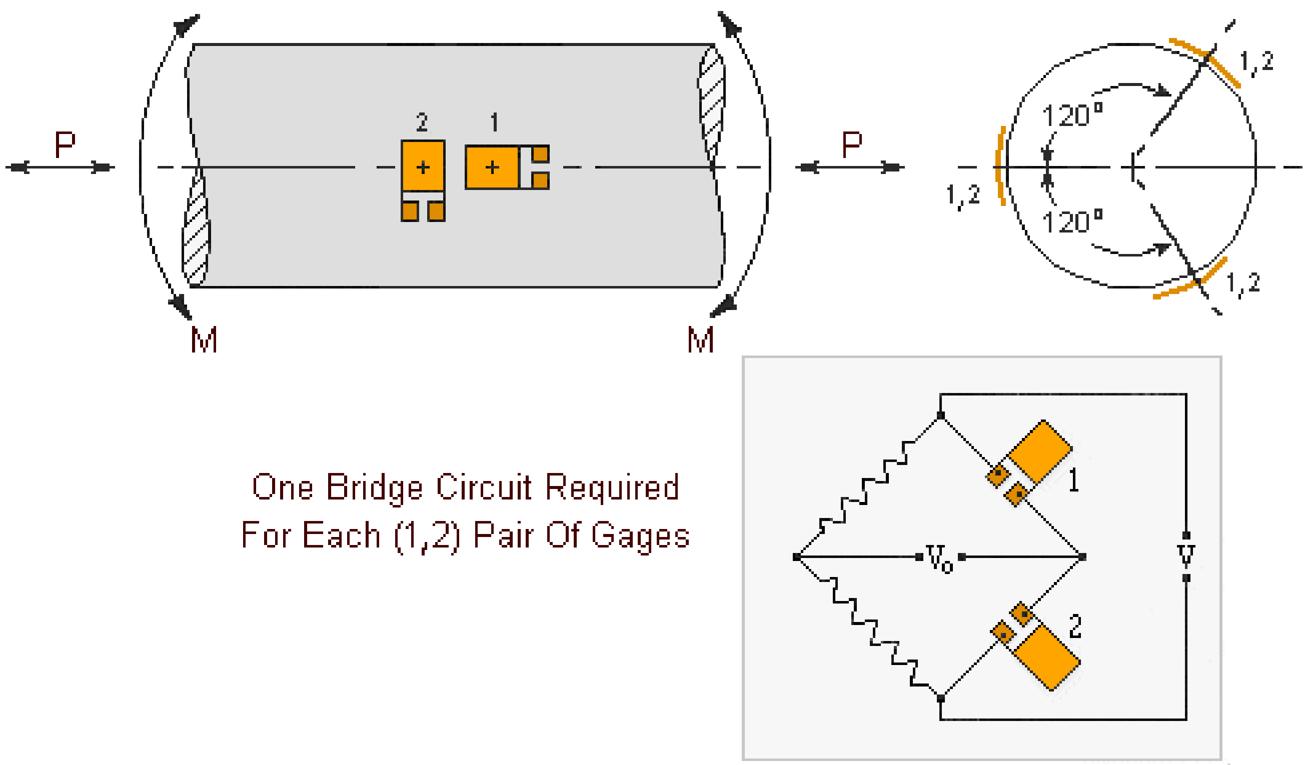
Separate Measurement of Combined Loads

In the treatment of the D-I-Y transducers described up to this point, techniques were illustrated not only for canceling the strain gage thermal output, but also for canceling load components other than the primary one which is the object of measurement. In each case, the cancellation of undesired signal components was achieved by taking advantage of Wheatstone bridge arithmetic -- the addition and subtraction of resistance changes in opposite and adjacent bridge arms, respectively. Sometimes, however, there is the need to simultaneously measure two mechanical variables, such as the axial force and bending moment on a shaft. This can be accomplished with multiple strain gages mounted on the shaft; but the strains must be measured individually for each gage (in a separate bridge circuit), and the load components obtained by subsequent data reduction.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)



Strain Gage And Circuit Arrangement For Separate Measurement Of Axial Force And Bending Moment

Consider, for example, the shaft illustrated above with an axial force and bending moment applied to it (or just an eccentric axial force, which would produce the same result). With three axially oriented strain gages installed at 120° intervals around the circumference of the shaft (at the same section), the shaft strain due only to the axial load component is:

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

$$\varepsilon_A = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{3}$$
(24)

And the axial force component can be obtained from:

$$P_A = \frac{\pi D^2}{4} E\varepsilon_A \times 10^{-6}$$
(25)

The maximum bending strain can be calculated from:

$$\varepsilon_B = \pm\frac{\sqrt{2}}{3}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}$$
(26)

And the applied bending moment from:

$$M_B = \frac{\pi D^3 E\varepsilon_B \times 10^{-6}}{32}$$
(27)

Finally, the angle from gage 1 to the principal bending plane is:

$$\theta = \tan^{-1}\frac{\sqrt{3}\ (\varepsilon_2 - \varepsilon_3)}{2\varepsilon_1 - \varepsilon_2 - \varepsilon_3}$$
(28)

It can be noticed that Eqs.(24), (26), and (28) directly parallel the data-reduction relationships for a delta rosette, except that in Eq.(28) the arctangent defines the actual angle from gage 1 to the principal plane of bending instead of the double angle. This characteristic of a circular cross-section shaft can be generalized to accommodate other gage locations. Thus, if the three axially oriented gages are spaced at 90° intervals around the shaft circumference, Eqs.(24), (26), and (28) can be replaced by the data-reduction relationships for a rectangular rosette. In this case,

(...continued)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

$$\varepsilon_A = \frac{\varepsilon_1 + \varepsilon_3}{2}$$
(29)

$$\varepsilon_B = \pm\frac{1}{\sqrt{2}}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2}$$
(30)

$$\theta = \tan^{-1}\frac{2\varepsilon_2 - \varepsilon_1 - \varepsilon_3}{\varepsilon_1 - \varepsilon_3}$$
(31)

Equations (25) and (27) are used as before to calculate the force and bending moment.

When the bending strains are calculated from Eqs.(26) and (28), there is a small error which can readily be corrected for if needed. The error occurs because the bending strain distribution across the shaft diameter is linear, but the width of the strain gage is wrapped around a cylinder. Because of this, the bending strain along the gage centerline is not precisely equal to the average bending strain sensed by all grid lines. The error is less than 1% if the gage width is less than 20% of the shaft diameter. When necessary, the maximum bending strain can be corrected as follows:

$$\varepsilon_B = \hat{\varepsilon}_B\frac{W/D}{\sin(W/D)}$$
(32)

Where: ε_B = corrected maximum bending strain

$\hat{\varepsilon}_B$ = maximum bending strain calculated from Eq.(26) or (28)

W = active width of gage grid
D = shaft diameter

No correction is needed for the calculated axial strain, ε_A.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

As described in the foregoing, the strain gages are employed in single-gage quarter-bridge circuits. This arrangement necessitates consideration of the thermal output of each gage if measurements are to be made under varying temperature conditions. One approach is to correct all strain measurements (before further data reduction) by subtracting the specified thermal output. The required data for this is supplied in each package of Micro-Measurements A-alloy and K-alloy strain gages. An alternative procedure is to install a tee rosette at each gage site, and connect the two gages of the rosette in a half-bridge circuit, as shown previously. Due to the algebraic subtraction of resistance changes in adjacent bridge arms, the thermal outputs of the gages will be canceled, and the bridge output signal augmented by a factor of (1 + *v*). The calculated axial and bending strains (ε_A and ε_B) should both be divided by (1 + *v*) to adjust for this signal augmentation.

For cases in which it is desired to measure axial force, bending moment, and torque in a shaft, a combination of transducer methods can be recommended. The axial force and bending moment can be obtained with three gages (in three separate bridge circuits) as described in this section. The torque measurement can best be made with four more gages, oriented at 45° to the shaft axis and connected in a full-bridge circuit as shown previously. With this combination, the axial-force and bending-moment gages will be insensitive to the torque; and the output of the torque bridge, if the gages are properly oriented and connected, will be insensitive to the axial force and bending moment.

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

Summary of Considerations for Improved Transducer Performance

Careful initial design, both mechanically and electrically, will contribute significantly to the accuracy, stability, and repeatability of a D-I-Y transducer. Whenever feasible, for example, the spring element should be designed as a monolithic or one-piece structure from the point of load application to the reaction. Bolted assemblies commonly lead to nonlinearity and hysteresis effects, and welded ones may also cause problems due to residual stresses in the heat-affected zones.

It is always good practice to design the spring element for ease of strain gage installation. Accurate location and orientation of the gages is critical to obtaining the operating characteristics described in the preceding sections. Obviously, precise gage positioning is more apt to be achieved if the gage installation site is flat, of sufficient size, and readily accessible. Preferably, the gage installation area should be subjected to uniform strain, and at the highest strain level in the spring element (but not above about 1500 $\mu\varepsilon$). If the strain magnitude varies over the gage site, it is advisable to install the gages with the solder-tab ends at the lowest strain level. Design consideration should also be given to assuring that the applied load acts, in both location and direction, as intended for the spring element configuration. Compensation methods for canceling off-axis load components are seldom 100 percent perfect; and the best transducer performance will be obtained from a design which minimizes such components from the beginning.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

If the transducer must operate under varying temperature conditions, it is important to note that cancellation of the strain gage thermal outputs within the bridge circuit is effective only when pairs of gages in adjacent arms of the circuit track each other thermally. Temperature differentials can be minimized by designing for a generous heat-conducting path between the two gages, and for thermally symmetric paths between the gages and any external sources of temperature change. Attention should also be given to the intra-bridge wiring which connects the gages to form the bridge circuit. For best results, the wires should be in thermal contact with the spring element, and the same length of wire should be in each arm of the circuit.

The strain gage excitation voltage should be low enough to avoid a sensible temperature rise in the gage due to self heating. For a given level of gage power, the rate of heat dissipation from the gage depends on the gage area, and on the mass of the spring structure under the gage, along with the thermal conductivity of the material. Recommendations for suitable power levels applicable to different spring materials are provided in Measurements Group Tech Note TN-502, *Optimizing Strain Gage Excitation Levels*. (Ref. 4) The bridge circuit output voltage at any particular power level can always be increased by increasing both the gage resistance and the excitation voltage. Because of this, gages of 350 ohms or higher resistance are preferable. When permitted by the physical shape of the spring element, and its strain distribution, it may sometimes be feasible to employ two gages electrically in series at each gage site.

(continued...)

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

(...continued)

It should also be recognized that a small change in transducer output normally accompanies any uniform change in the temperature of the spring element. This occurs because both the elastic modulus of the spring material and the gage factor of the strain gage vary with temperature. With the usual spring materials, the elastic modulus tends to decrease by about 1% to 3% for a temperature rise of 100°F (55°C), causing an increase in strain and transducer output for the same load. In the case of constantan strain gages, for instance, the gage factor increases by approximately 0.5% for the same temperature rise, further increasing the transducer output.

In precision commercial transducers this effect, known as "span shift with temperature" is often compensated for by introducing a temperature-sensitive resistor (e.g., nickel) in one of the leads supplying power to the Wheatstone bridge circuit. As the temperature rises, the compensating resistance increases, reducing the supply voltage to the bridge circuit accordingly. The detailed procedure for accurate compensation of span shift by this method cannot be adequately treated here, but is described in References (Ref. 5) and (Ref. 6), along with additional circuit refinements commonly found in commercial transducers.

Transducer manufacturers typically employ not only circuit refinements but also quite sophisticated mechanical design of the spring elements to achieve their desired performance objectives. The latter may include, for instance, an accuracy specification of 0.1% or better. The relatively simple spring configurations and circuit arrangements given in this Tech Note should yield transducer accuracies of perhaps 1% to 5% (after calibration), depending on the transducer operating conditions and on the care with which the recommendations provided here are implemented.

TECHNOLOGY

Measurement of Force, Torque, and Other Mechanical Variables With Strain Gages

REFERENCES

1. "Strain Gage Based Transducers", Measurements Group, Inc., 1988.

2. Timoshenko, S. and J. N. Goodier, *Theory of Elasticity*, McGraw-Hill Book Co., 1951, pp. 275-278.

3. "Design Considerations for Diaphragm Pressure Transducers", Tech Note TN-510, Measurements Group, Inc., 1982.

4. "Optimizing Strain Gage Excitation Levels", Tech Note TN-502, Measurements Group, Inc., 1979.

5. Dorsey, J., "Homegrown Strain-Gage Transducers", *Experimental Mechanics*, Vol. 17, No. 7, July, 1977, pp. 255-260.*

6. Dorsey, J. , "Linearization of Transducer Compensation", ISA, ISBN 87664-362-4.*

---

* Available on request from Measurements Group, Inc.