



## 8<sup>th</sup> Lecture

# Impact

### Contents:

8.1 – Dynamic Coefficient	2
8.2 – Longitudinal Impact of a Weight on a Bar	4
8.3 – Strain Gauge for Impact Phenomena	6

## 8.1 – Dynamic Coefficient

An impact phenomenon generates at a sudden change of velocities of touching bodies, systems or their parts.

Let us show an engineering solution of simple cases of an impact of a moving body (**impacting body**) on a stagnant body or system (**impacted body**). For the solution let us consider the following simplifying presumptions:

1. The impacting body is absolutely rigid.
2. The impacting body has one degree of freedom and its generalized displacements are proportional to the corresponding force effects at static and dynamic actions
3. The impact is inelastic so that no separation of the impacting and the impacted bodies occurs; nevertheless, all the deformations of the impacted body are elastic.
4. Character of deformation of the impacted body is the same as at static loading by corresponding force action at the point of impact and in the impact direction.
5. The velocity of the impacted body is small in comparison with the propagation speed of the impact waves in material and the time of the impact is considerably longer than the time of the impact wave spreading all over the volume of the impacted body.
6. On the above-given conditions it is possible to determine approximately dynamic forces  $F_{dyn}$ , stress  $s_{dyn}$  and displacements  $d_{dyn}$  in the impacted body from the relations

$$\left. \begin{aligned} F_{dyn} &= k_{dyn} F_{st} \\ S_{dyn} &= k_{dyn} S_{st} \\ d_{dyn} &= k_{dyn} d_{st} \end{aligned} \right\}, \quad (8.1)$$

where  $F$ ,  $s$  and  $d$  are determined at the static force acting on the impacted body, and that at the impact point and in the impact direction; „ $k_{dyn}$ “ is a dimensionless dynamic coefficient.

If the impacting body of the weight  $Q$  moves at the moment of collision with the impacted body of the weight  $Q_0$  at the speed  $v_0$  in the direction of gravity (see Fig.8.1) and causes displacements in the elements of the impacted body, the dynamic coefficient  $k_{dyn}$  can be determined by the following steps.

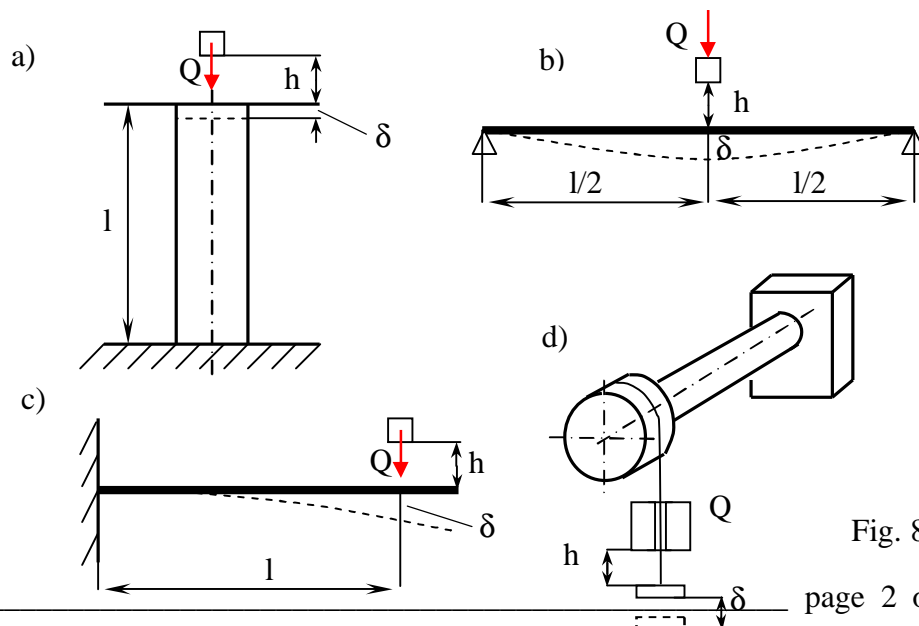


Fig. 8.1

Let us consider systems of bodies given in the Fig.8.1. These structures are loaded by the weight  $Q$  falling from the height  $h$ .

Let us suppose that the weight of the impacted body can be neglected in the calculation.

Let us denominate  $d_{dyn}$  maximal displacement of the investigated system in the direction of the loading  $Q$ . At the moment when the deformation of the structure reaches its maximum, all the potential energy of the weight is accumulated in the impacted body, i.e.

$$Q \cdot (h + d_{dyn}) = U \quad , \quad (8.2)$$

where  $U$  is deformation energy accumulated in the impacted body. By means of the relations (8.1) it can be written:

$$U = \frac{1}{2} Q_{dyn} \cdot d_{dyn} = \frac{1}{2} k_{dyn} \cdot Q \cdot k_{dyn} \cdot d_{st} = \frac{1}{2} k_{dyn}^2 \cdot Q \cdot d_{st} \quad . \quad (8.3)$$

Having substituted the previous expression into the equation 8.2, and following the equation conversion we shall get:

$$k_{dyn}^2 - 2k_{dyn} - 2 \frac{h}{d_{st}} = 0 \quad .$$

Herefrom we shall get an expression for the dynamical coefficient  $k_{dyn}$  for the problems similar to those in the Fig.8.1, i.e.

$$k_{dyn} = 1 + \sqrt{1 + 2 \frac{h}{d_{st}}} \quad . \quad (8.4)$$

At the free fall of the weight  $Q$  from the height  $h$ , the impact velocity is given by the relation

$$v = \sqrt{2 g h} \quad ,$$

where  $g$  is acceleration of gravity. Herefrom

$$2h = \frac{v^2}{g} \quad .$$

Having substituted it in the relation 8.4 we shall get

$$k_{dyn} = 1 + \sqrt{1 + \frac{v^2}{g \cdot d_{st}}} \quad . \quad (8.5)$$

**Fall from the zero height**

In case  $h=0$  , the dynamic coefficient according to the relation (8.4) is.

$$k_{dyn} = 1 + \sqrt{1+0} = 2 .$$

It is obvious that at a sudden loading the stress is double in comparison with the state of stress at slow loading.

**8.2 – Longitudinal Impact of a Weight on a Bar**

Let us consider a bar of the length  $l$  and of the constant cross-section  $S$  (see Fig. 8.2a). Let the weight  $Q$  fall on the bar from the height  $h$ .

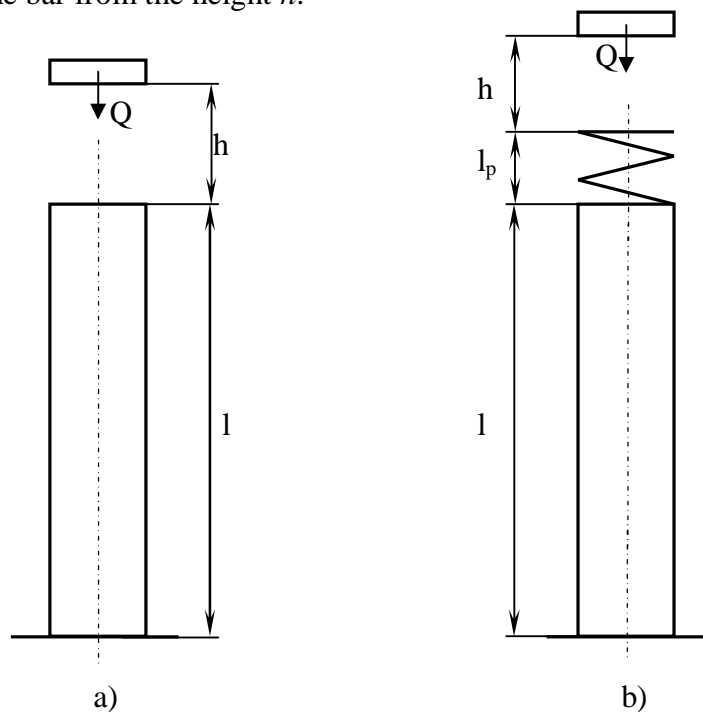


Fig. 8.2

The deformation of the bar of the length  $l$  at static loading by the weight  $Q$  is given by the relation

$$d_{st} = \frac{Q \cdot l}{E \cdot S} ,$$

where  $E$  is modulus of elasticity of the bar material. The dynamic coefficient is then equal:

$$k_{dyn} = 1 + \sqrt{1 + \frac{2hES}{Ql}} .$$

Maximal stress at the impact is

$$s_{\max} = -k_{dyn} \cdot s_{st} = -\frac{Q}{S} \left( 1 + \sqrt{1 + \frac{2hES}{Ql}} \right).$$

Maximal relative deformation at the impact is

$$e_{\max} = \frac{s_{\max}}{E} = -\frac{Q}{ES} \left( 1 + \sqrt{1 + \frac{2hES}{Ql}} \right).$$

**For reducing the loading** of the bar at the weight impact, a **spring of the compliance „c“ is inserted between the weight and the bar** (see Fig. 8.2b).

Let us consider a densely wound spring. Then the compliance of this spring is:

$$c = \frac{8D^3 n}{G d^4},$$

where  $D$  is the spring diameter,

$d$  is the wire diameter,

$n$  is number of coils,

$G$  is shear modulus of the spring wire.

The spring deformation at the static loading by the weight  $Q$  is

$$d_{P_{st}} = c \cdot Q = \frac{8D^3 \cdot n}{G \cdot d^4} \cdot Q.$$

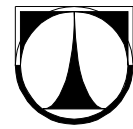
The total deformation at the gradual loading by the weight  $Q$  is given by the relation

$$d_{Calk_{st}} = Q \left( \frac{8D^3 n}{G d^4} + \frac{l}{ES} \right).$$

The dynamical coefficient is

$$k_{dyn} = 1 + \sqrt{1 + \frac{2h}{Q \left( \frac{8D^3 n}{G d^4} + \frac{l}{ES} \right)}}.$$

It is obvious from the previous relation that the influence of the spring considerably decreases the dynamical coefficient and thus also the bar stress and deformation.



---

### 8.3 – Strain Gauge for Impact Phenomena

Measurement of the impact phenomena is an entirely specific type of dynamic tests.

This type of measurement occurs during the investigation of the rapid crack propagation, processes during dynamic forming, in the vehicle development etc. Typical for all the impact phenomena are short times of durations, un-reproducibility of the investigated phenomena and presence of high frequencies in the recorded signals. Therefore the recording and registration methods of the impact phenomena have to be adapted to these facts. It is necessary for the applied sensors to have good frequency properties, i.e. the highest possible natural resonance frequency and the lowest possible minimal transferred frequency.