7th Lecture

Measurement of Mechanical Oscillations

Contents:

7.1 – Introduction

A periodical character of work of most machines results in periodical loading and deforming of both individual parts of the machines and the constructions that create the basements of these machines. For measuring of mechanical oscillations often such sensors are use, which can be considered, from the point of view of mechanics, to be systems with inertia mass, spring and eventual viscous damper.

In the present lecture let us focus on systems without damping.

7.2 – Inertia System with Spiral Springs

In the fig. 7.1 there are illustrated the systems with linear cylindrical spiral springs the resonance frequency of which is:

$$f_0 = \frac{1}{2p}\sqrt{\frac{k}{m + \frac{1}{3} m_p}} \qquad (7.1)$$

where k is the spring constant (stiffness) and m_p is the spring weight. The constant k (stiffness) of the spiral spring can be determined from the relation:

$$k = \frac{G \cdot d^4}{8n \cdot D^3} \qquad (7.2)$$

where G is modulus of elasticity in torsion,

d is the spring wire diameter,

n is a number of active spring coils,

D is the spring diameter.

In the figure 7.1 b there are 2 springs parallel-connected to the mass *m*. The resulting stiffness is given by the relation:

$$k = k_1 + k_2 \quad . \qquad (7.3)$$

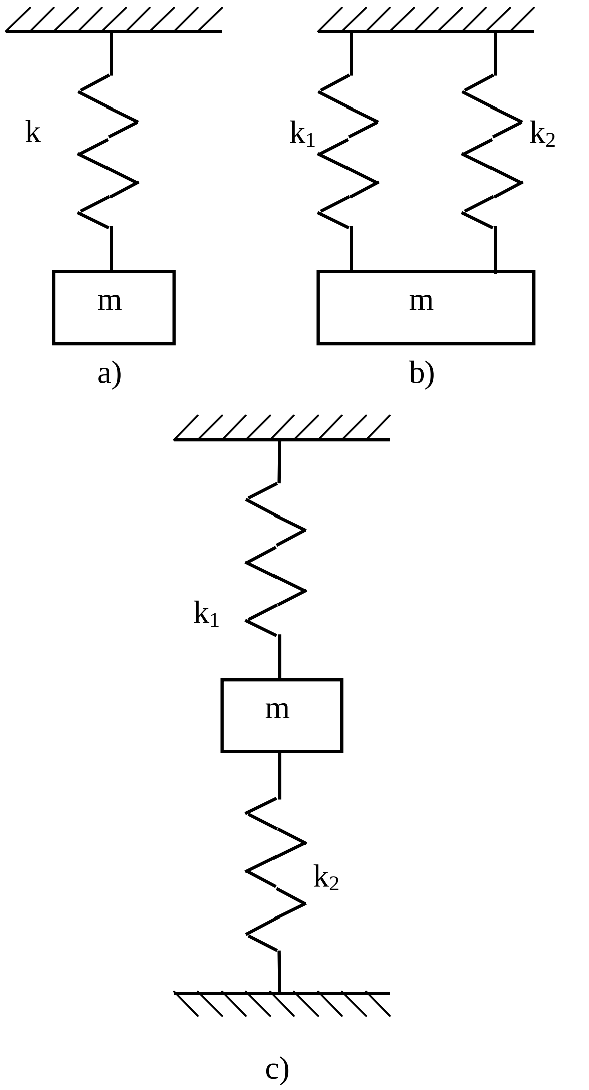

Fig. 7.1

In the Fig.7.1 c there are given 2 springs, series-connected to the mass *m*. The resulting stiffness is given by the relation: (7.4)

$$k = \frac{k_1 \cdot k_2}{k_1 + k_2}$$

The resonance frequency of the spiral spring itself is:

$$f_0 = \frac{d}{4pn \cdot D^2} \sqrt{\frac{G \cdot g}{2r}} \quad , \tag{7.5}$$

where g is gravitation acceleration

r is specific weight of the spring material.

7.3 – Inertia System with One Flat Spring

The measuring system of reed-type frequency meters is created by a beam with one fixed end with an additional inertia mass at its free end (see Fig.7.2). Now let us show finding the natural frequency of the beam showed in the fig. 7.2. Let us neglect all damping during the problem solving.

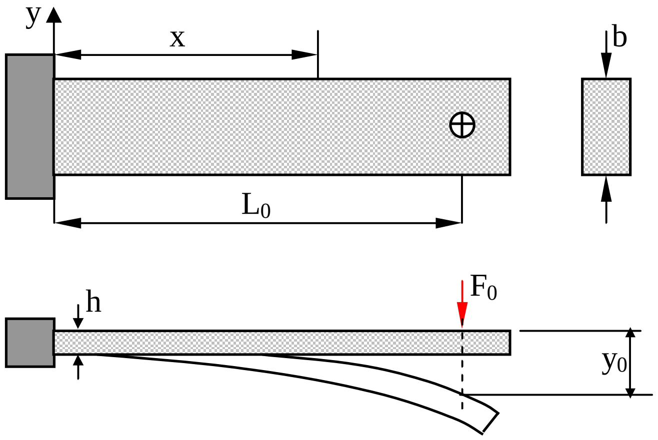

Fig. 7.2

Bending momentum *M* at the point *x* is given by the relation:

$$M(x) = -F_0 \cdot L_0 + F_0 \cdot x \tag{7.6}$$

where L_0 is the distance of the application point of the force F_0 from the point of the beam fixation. The beam deflection caused by the force F_0 is:

$$EJ\frac{d^2y}{dx^2} = -F_0 \cdot L_0 + F_0 \cdot x \tag{7.7}$$

Via integration and considering the boundary conditions for the point of fixation we shall get the equation of a deflection line in the shape:

$$y = \frac{F_0}{EJ} \cdot \left(-\frac{L_0 x^2}{2} + \frac{x^3}{6} \right) \tag{7.8}$$

Displacement y_0 at the point of application of the force F_0 $(x=L_0)$ is given by the relation

$$y_0 = \frac{F_0 \cdot L_0^{\ 3}}{3EJ} \tag{7.9}$$

Considering the inertia moment J for the cross-section of the investigated beam being

$$J = \frac{1}{12} b h^3 \ , \tag{7.10}$$

then

$$F_0 = \frac{E\, b\, h^3}{4 \cdot L_0^{\ 3}} \cdot y_0 \ , \tag{7.11}$$

i.e.

$$F_0 = k \cdot y_0 \ ,$$

where

$$k = \frac{E\, b\, h^3}{4 L_0^{\ 3}} \tag{7.12}$$

is reduced rigidity of the considered fixed beam at the point of application of the force F_0.

Maximal potential energy accumulated in the given beam is for y_0 Î $<0;y_{max}>$ given by the relation:

$$U_{max} = \int_0^{y_{max}} F_0 d\, y_0 = \int_0^{y_{max}} \frac{E\, b\, h^3}{4 L_0^{\ 3}} \cdot y_0 \cdot dy_0 = \frac{E\, b\, h^3}{8 L_0^{\ 3}} \cdot y^2{}_{max} \tag{7.13}$$

The kinetic energy linked with the vibrating beam is given by the mass <u>*m*</u> placed at the location L_0 and the velocity $v = dy_0 / dt$, of the mass <u>*m*</u> motion.

I.e.

$$T = \frac{1}{2} m \left(\frac{dy_0}{dt} \right)^2 . \qquad (7.14)$$

In case of free harmonic motion it can be written:

$$y_0 = y_{\max} \cdot \cos(wt)$$

$$\frac{dy_0}{dt} = -w\, y_{\max} \cdot \sin(wt) \qquad (7.15)$$

The maximal velocity is reached twice in every cycle and that at the angle *p/2* and *3p/2*.

Thus

$$v_{\max} = \pm w \cdot y_{\max} \quad , \qquad (7.16)$$

where *w* is angular velocity in radians. The oscillation frequency is:

$$f = \frac{w}{2p} \; .$$

It can be written:

$$v_{\max} = \pm 2pf \cdot y_{\max} \quad , \qquad (7.17)$$

$$T_{\max} = \frac{1}{2} m v^2{}_{\max} = \frac{1}{2} m (2p \;\; f \;\; y_{\max})^2 \; . \qquad (7.18)$$

It is possible to imagine the oscillations as a repeated transfer of certain amount of energy from the mass *m* to the deformed beam. When the beam deflection is at its maximum, the mass comes to halt for a moment and the kinetic energy ceases. At that moment all the system energy is stored in the spring in the form of potential energy. When the mass passes through the point $y_0=0$, it has the maximal velocity and its motion concentrates all the system energy as the beam is not deformed. At the other points of the cycle the ratio of the kinetic and potential energies varies, but their sum is always constant. If there are no losses during the beam oscillation, then it is valid:

$$U_{\max} = T_{\max} \; .$$

I.e.

$$\frac{E\, b\, h^3}{8 L_0{}^3} y^2{}_{\max} = \frac{1}{2} m (2p \cdot f \cdot y_{\max})^2 \; .$$

Herefrom, the natural frequency is:

$$f = \frac{1}{2p} \cdot \sqrt{\frac{E\,b\,h^3}{4mL_0^{\;3}}} = \frac{1}{2p} \cdot \sqrt{\frac{k_r}{m}} \quad . \qquad (7.19)$$

The period *T* of vibrations is:

$$T = \frac{2p}{w} = \frac{1}{f} = 2p \cdot \frac{1}{\sqrt{\frac{k_r}{m}}} \quad . \qquad (7.20)$$

If the motion of the mass *m* at the point $x = L_0$ is harmonic and there are no energy losses, then the relation (7.19) or (7.20) shows that the fixed beam oscillates with the frequency determined by its ***modulus of elasticity*** (material property), ***geometry*** (b, h, L_0) and ***applied mass.***.