

5th Lecture

Plane State of Stress – Strain-gauge Rose

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5.1 – Introduction

When measuring with the resistance strain gauges, the sensor reacts to a relative prolongation in the direction of strain gauge axis and relatively slightly also to the deformation perpendicular to this direction. Thus the sensor does not react to the shearing strain nor the deformation caused by shear. The measured value of the resistance change DR/R or directly relative prolongation e must be usually converted to stress, which is crucial for examination of stressing of the parts.

When measuring uniaxial stress (see the 2nd and 3rd Lecture), the sensor is oriented in the direction of stress.

Thus it is valid

$$s = e \cdot E,$$

where s is the searched stress [MPa]

e is measured relative prolongation

E is modulus of elasticity of material [MPa]

When investigating biaxial state of stress with known directions of principal stress (Lecture 4), it is necessary to measure deformation in both these directions. For this purpose, two sensors or a strain-gauge cross with two perpendicular windings are applied.

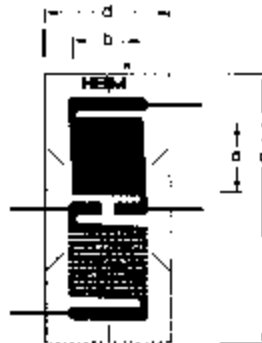


Fig. 5.1

From the magnitudes of both these deformations ϵ_1 and ϵ_2 it is possible to determine the magnitudes of the principal stresses s_1 and s_2 .

5.2 – Plane State of Stress

When investigating plane state of stress with unknown directions of the principal stresses, it is not possible to direct distinctively the strain-gauge cross into the directions of maximal and minimal stresses.

Similarly as in the Mohr's circle for stress (4th Lecture), a relation between deformation e and shearing strain g at a certain point on the surface can be recorded by means of the Mohr's circle for deformation.

In the Fig. 5.2 there is a Mohr's circle for deformation. The deformation e_a is deflected from the maximal strain by the angle a and thus by the angle $2a$ in the same sense in the Mohr's circle.

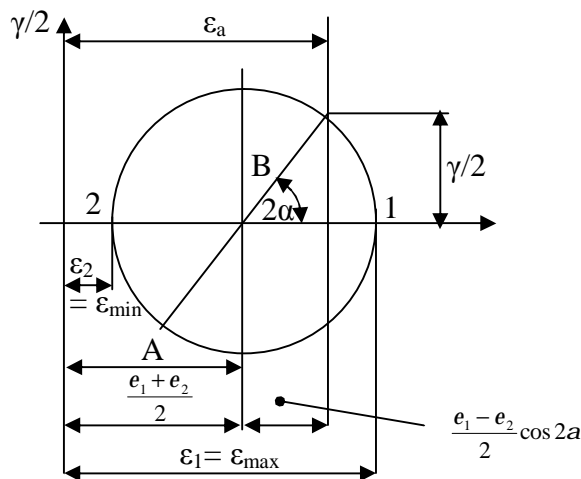


Fig. 5.2

For e_a the following relation is valid

$$e_a = \frac{e_{\max} + e_{\min}}{2} + \frac{e_{\max} - e_{\min}}{2} \cos 2a$$

Let us further introduce a denomination

$$A = \frac{e_1 + e_2}{2} \quad B = \frac{e_1 - e_2}{2}$$

Then we shall get the equation for e_a in the shape

$$e_1 = A + B \quad e_2 = A - B$$

As it is obvious from the Fig. 5.2, the value A represents a distance of the centre of the Mohr's circle of deformation from the beginning and the value B represents the radius of this circle.

5.3 – Strain-gauge Rose

In order to determine unknown values of e_1 , e_2 and deflection a of the sensor a from the direction of the principal stresses, it is necessary to know deformations at least in three directions e_a , e_b and e_c , diverted from the direction of the principal stresses or, as the case may be, deformation e_l by the angle a , b and g .

These directions can be on principle chosen fully arbitrarily. However, regarding the computation simplification, it is useful to choose them under a certain angle. Therefore so call strain-gauge roses are made, containing three or four measuring windings on a common basis. This ensures both a precise mutual position of individual windings and makes the installation easier.

The following types of roses are used in experimental stress analysis:

a) Right-angled rose

This rose (Fig. 5.3) is the most common type and is used in cases, where the directions of the principle stresses can be at least approximately estimated. The relations for maximal and minimal deformation and stress at a given point of installation of the right-angled rose are as follows:

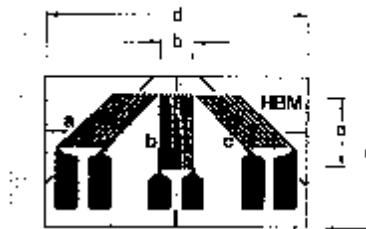


Fig. 5.3 – Denomination of the rose HBM – RY81

Maximal and minimal deformation:

$$e_{\max} = \frac{1}{2}(e_a + e_b) + \frac{1}{\sqrt{2}} \cdot \sqrt{(e_a - e_b)^2 + (e_b - e_c)^2}$$

$$e_{\min} = \frac{1}{2}(e_a + e_b) - \frac{1}{\sqrt{2}} \cdot \sqrt{(e_a - e_b)^2 + (e_b - e_c)^2}$$

Maximal and minimal stress:

$$s_{\max} = \frac{E}{2} \left\{ \frac{1}{(1-u)} \cdot (e_a + e_c) + \frac{1}{1+u} \cdot \sqrt{(e_a - e_c)^2 + [2e_b - (e_a + e_c)]^2} \right\}$$

$$s_{\min} = \frac{E}{2} \left\{ \frac{1}{(1-u)} \cdot (e_a + e_c) - \frac{1}{1+u} \cdot \sqrt{(e_a - e_c)^2 + [2e_b - (e_a + e_c)]^2} \right\}$$

Maximal shear stress:

$$t_{\max} = \frac{1}{2}(s_{\max} - s_{\min}) = \frac{E}{2(1+u)} \sqrt{(e_a - e_c)^2 + [2e_b - (e_a + e_c)]^2}$$

Angle between the axis \underline{a} of the strain-gauge and the direction of the maximal stress:

$$a = \frac{1}{2} \operatorname{arc\,tg} \cdot \left[\frac{2e_b - (e_b + e_c)}{e_a - e_c} \right]$$

b) Equilateral rose with the angle 60°

This rose is used in the locations where the directions of the maximal and minimal stresses cannot be estimated. The rose is often called a *DELTA* – rose (see Fig. 5.4).

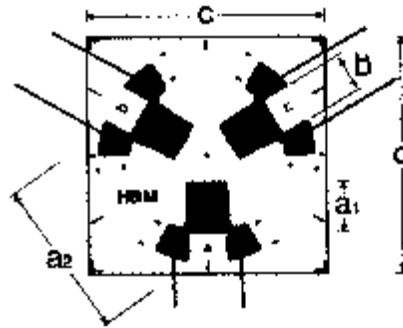


Fig. 5.4 – Denomination of the rose HBM – RY 41

Maximal and minimal deformation:

$$e_{\max} = \frac{1}{3}(e_a + e_b + e_c) + \frac{\sqrt{2}}{3} \cdot \sqrt{(e_a - e_b)^2 + (e_b - e_c)^2 + (e_c - e_a)^2}$$

$$e_{\min} = \frac{1}{3}(e_a + e_b + e_c) - \frac{\sqrt{2}}{3} \cdot \sqrt{(e_a - e_b)^2 + (e_b - e_c)^2 + (e_c - e_a)^2}$$

Maximal and minimal stress:

$$s_{\max} = E \left\{ \frac{1}{3(1-u)} \cdot (e_a + e_b + e_c) + \frac{1}{1+u} \cdot \sqrt{\left(e_a - \frac{e_a + e_b + e_c}{3} \right)^2 + \left(\frac{e_c - e_b}{\sqrt{3}} \right)^2} \right\}$$

$$s_{\min} = E \left\{ \frac{1}{3(1-u)} \cdot (e_a + e_b + e_c) - \frac{1}{1+u} \cdot \sqrt{\left(e_a - \frac{e_a + e_b + e_c}{3} \right)^2 + \left(\frac{e_c - e_b}{\sqrt{3}} \right)^2} \right\}$$

Maximal shear stress:

$$t_{\max} = \frac{E}{2(1+u)} \sqrt{\left(e_a - \frac{e_a + e_b + e_c}{3} \right)^2 + \left[\frac{(e_c - e_b)}{\sqrt{3}} \right]^2}$$

Angle between the axis a of the strain gauge and the direction of the maximal stress:

$$a = \frac{1}{2} \operatorname{arc\,tg} \cdot \left[\frac{\frac{1}{\sqrt{3}}(e_c - e_b)}{e_a - \frac{e_a + e_b + e_c}{3}} \right]$$

5.4 – Analysis of Admissibility of the Measured Values of Deformations or Evaluated Stresses

Having investigated experimentally the state of stress we cannot content with a mere statement of the magnitude of the measured deformation but the existing magnitude has to be examined in relation to the reliable function of the investigated part.

A limit state of the machinery part or construction is the state when the required function is lost. Such a limit state can be an occurrence of:

- ∅ excessive plastic deformations
- ∅ a fatigue fracture
- ∅ a sudden brittle fracture
- ∅ elastic instability

For each limit state a certain function f can be defined, which takes the limit value f_m when reaching the limit state.

The limit state occurs, when: $f=f_m$

An acceptable working state is given by the condition: $f < f_m$ or $f = f_m/k$, where $k > 1$ is the safety factor of the working state compared with the limit state.

The oldest and so far the most widespread hypothesis for the machine parts dimensioning is the hypothesis, that the limit value f_m is only a material quantity that can be determined by means of uniaxial tension or pressure.

By the selection of the above-mentioned function f were in the past formulated so called strength hypotheses or strength conditions. Practically two groups of strength conditions are distinguished for the machine parts, namely for

- a) ductile materials
- b) brittle materials

Strength condition of the maximal shear stress – Guest's (Tresca's) hypothesis

This condition is suitable for investigation of the occurrence of excessive plastic deformations.

It is valid:

$$S_{\max} - S_{\min} = S_k ,$$

$$f = S_{\max} - S_{\min} ,$$

$$f_m = S_k ,$$

where S_k is the yield stress.

HMH strength condition

HMH hypothesis is also suitable for examination of the plastic deformation occurrence in the investigated part. For the biaxial state of stress it is valid:

$$\sqrt{s_{\max}^2 - s_{\max} s_{\min} + s_{\min}^2} = s_k,$$

i.e.

$$f = \sqrt{s_{\max}^2 - s_{\max} s_{\min} + s_{\min}^2},$$

$$f_m = s_k.$$

Strength condition according to Saint Venant

This condition comes out of the presumption that the decisive quantity is the maximal relative prolongation ε_{\max} . The condition is suitable for brittle materials.