1a) Ohybový moment, posouvající síla a průhyb vybraných nosníků s konstantním průřezem a různě zatížených (staticky určité případy)

P .. osamělá síla

R_1, R_2 reakce

w ... spojité zatížení $[w] = N/m$

W .. celkové spojité zatížení na nosník

l ... délka nosníku

x ... vzdálenost od podpory (osa x)

E ... modul pružnosti

I ... moment setrvačnosti

V_x .. posouvající síla v místě x

V ... maximální posouv. síla

M_x ... ohybový moment v místě x

M ... maximální ohyb. moment

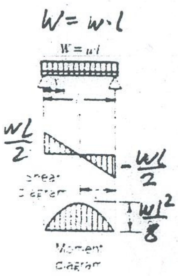
y ... maximální průhyb

Table 22.5 Bending Moment, Vertical Shear, and Deflection of Beams of Uniform Cross Section under Various Conditions of Loading

P = concentrated loads, lb
R_1, R_2 = reactions, lb
w = uniform load per unit of length, lb per in.
W = total uniform load on beam, lb
l = length of beam, in
x = distance from support to any section, in
E = modulus of elasticity, psi
I = moment of inertia, in.^{4} (J_y)
V_x = vertical shear at any section, lb ($T_z = T$)
V = maximum vertical shear, lb
M_x = bending moment at any section, lb-in. (M_o, M_y)
M = maximum bending moment, lb-in.
y = maximum deflection, in. (w_{max})

Simple Beam—Uniform Load

$W = w \cdot l$



$R_1 = R_2 = \frac{wl}{2}$

$V_x = \frac{wl}{2} - wx$

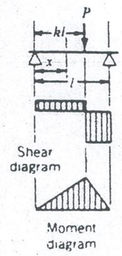
$V = \pm\frac{wl}{2}\left(\text{when }\begin{cases} x = 0 \\ x = l\end{cases}\right)$

$M_x = \frac{wlx}{2} - \frac{wx^2}{2}$

$M = \frac{wl^2}{8}\left(\text{when } x = \frac{l}{2}\right)$

$y = \frac{5Wl^3}{384EI}$ (at center of span)

Simple Beam—Concentrated Load at Any Point

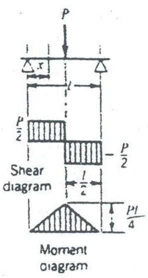


$R_1 = P(1 - k)$
$R_2 = Pk$
$V_x = R_1$ (when $x < kl$)
$= R_2$ (when $x > kl$)
$V = P(1 - k)$ (when $k < 0.5$)
$= -Pk$ (when $k > 0.5$)
$M_x = Px(1 - k)$ (when $x < kl$)
$= Pk(l - x)$ (when $x > kl$)
$M = Pkl(1 - k)$ (at point of load)

$y = \frac{Pl^3}{3EI}(1 - k) \times (2/3k - 1/3k^2)^{3/2}$

(at $x = l\sqrt{2/3k - 1/3k^2}$)

Simple Beam—Concentrated Load at Center



$R_1 = R_2 = \frac{P}{2}$

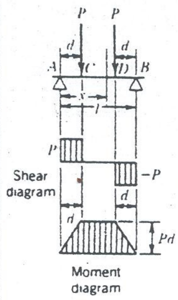
$V_x = V = \pm\frac{P}{2}$

$M_x = \frac{Px}{2}$

$M = \frac{Pl}{4}\left(\text{when } x = \frac{l}{2}\right)$

$y = \frac{Pl^3}{48EI}$ (at center of span)

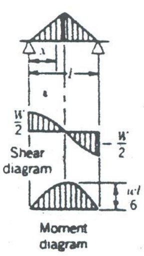
Simple Beam—Two Equal Concentrated Loads at Equal Distances from Supports



$R_1 = R_2 = P$
$V_x = P$ for AC
$= 0$ for CD
$= -P$ for DB
$V = \pm P$
$M_x = Px$ for AC
$= Pd$ for CD
$= P(l - x)$ for DB
$M = Pd$

$y = \frac{Pd}{24EI}(3l^2 - 4d^2)$ (at center of span)

Simple Beam—Load Increasing Uniformly from Supports to Center of Span



$R_1 = R_2 = \frac{W}{2}$

$V_x = W\left(\frac{1}{2} - \frac{2x^2}{l^2}\right)\left(\text{when } x < \frac{l}{2}\right)$

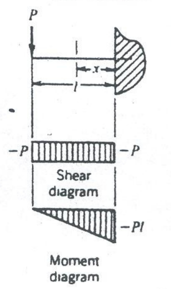
$V = \pm\frac{W}{2}$ (at supports)

$M_x = Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$

$M = \frac{Wl}{6}$ (at center of span)

$y = \frac{Wl^3}{60EI}$ (at center of span)

Cantilever Beam—Load Concentrated at Free End



$R = P$

$V_x = V = -P$

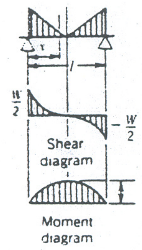
$M_x = -P(l - x)$

$M = -Pl$(when $x = 0$)

$y = \frac{Pl^3}{3EI}$

Table 22.5 *(Continued)*

Simple Beam — Load Increasing Uniformly from Center to Supports



$$R_1 = R_2 = \frac{W}{2}$$

$$V_x = -W\left(\frac{2x}{l} - \frac{2x^2}{l^2} - \frac{1}{2}\right) \quad \left(\text{when } x < \frac{l}{2}\right)$$

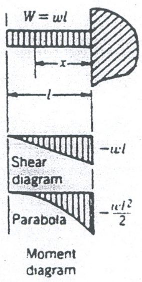
$$V = \pm\frac{W}{2}$$

$$M_x = Wx\left(\frac{1}{2} - \frac{x}{l} + \frac{2}{3}\frac{x^2}{l^2}\right) \quad \left(\text{when } x < \frac{l}{2}\right)$$

$$M = \frac{Wl}{12} \text{ (at center of span)}$$

$$y = \frac{3}{320}\frac{Wl^3}{EI} \text{ (at center of span)}$$

Cantilever Beam—Uniform Load



$$R = W = wl$$

$$V_x = -w(l - x)$$

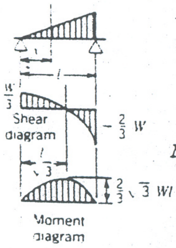
$$V = -wl \text{ (when } x = 0)$$

$$M_x = -w(l - x)\left(\frac{l - x}{2}\right)$$

$$M = -\frac{wl^2}{2} \text{ (when } x = 0)$$

$$y = \frac{Wl^3}{8EI}$$

Simple Beam — Load Increasing Uniformly from One Support to the Other



$$R_1 = \frac{W}{3}; \quad R_2 = \frac{2}{3}W$$

$$V_x = W\left(\frac{1}{3} - \frac{x^2}{l^2}\right)$$

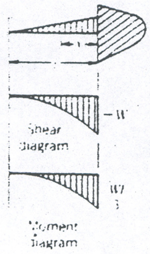
$$V = -\frac{2}{3}W \text{ (when } x = l)$$

$$M_x = \frac{Wx}{3}\left(1 - \frac{x^2}{l^2}\right)$$

$$M = \frac{2}{9\sqrt{3}}Wl \quad \left(\text{when } x = \frac{l}{\sqrt{3}}\right)$$

$$y = \frac{0.01304}{EI}Wl^3$$

Cantilever Beam—Load Increasing Uniformly from Free End to Support



$$R = W$$

$$V_x = -W\frac{(l - x)^2}{l^2}$$

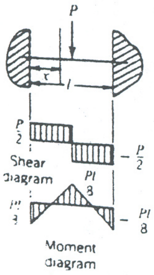
$$V = -W \text{ (when } x = 0)$$

$$M_x = -\frac{W}{3}\frac{(l - x)^3}{l^2}$$

$$M = -\frac{Wl}{3} \text{ (when } x = 0)$$

$$y = \frac{Wl^3}{15EI}$$

Fixed Beam — Concentrated Load at Center of Span



$$R_1 = R_2 = \frac{P}{2}$$

$$V_x = V = \pm\frac{P}{2}$$

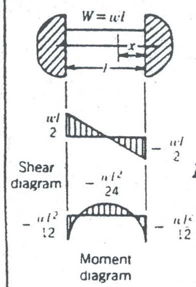
$$M_x = P\left(\frac{x}{2} - \frac{l}{8}\right)$$

$$M_x = -\frac{Pl}{8}\left(\text{when } \begin{cases} x = 0 \\ x = l \end{cases}\right)$$

$$M = +\frac{Pl}{8} \text{ (at center of span)}$$

$$y = \frac{Wl^3}{192EI}$$

Fixed Beam—Uniform Load



$$R_1 = R_2 = \frac{wl}{2} = \frac{W}{2}$$

$$V_x = \frac{wl}{2} - wx$$

$$V = \pm\frac{wl}{2} \text{ (at ends)}$$

$$M_x = -\frac{wl^2}{2}\left(\frac{1}{6} - \frac{x}{l} + \frac{x^2}{l^2}\right)$$

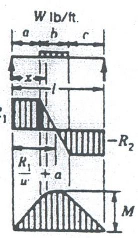
$$M = -1/_{12}\, wl^2 \quad \left(\text{when } \begin{cases} x = 0 \\ x = l \end{cases}\right)$$

$$M = \frac{wl^2}{24}\left(\text{when } x = \frac{l}{2}\right)$$

$$y = \frac{Wl^3}{384EI}$$

Table 22.5 (*Continued*)

Simple Beam—Distributed Load over Part of Beam



$$R_1 = \frac{wb(2c+b)}{2l}$$

$$R_2 = \frac{wb(2a+b)}{2l}$$

$$V_x = \frac{wb(2c+b)}{2l} - w(x-a)$$

$V = R_1$ (when $a < c$)

$= R_2$ (when $a > c$)

$$M_x = \frac{wbx(2c+b)}{2l} \text{ (when } x < a)$$

$$= R_1x - \frac{w(x-a)^2}{2}$$

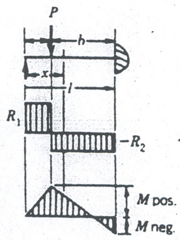
(when $a < x < a+b$)

$= R_2(l-x)$

(when $l - x < c$)

$$M = \frac{wb(2c+b)[4al+b(2c+b)]}{8l^2}$$

Beam Supported at One End, Fixed at Other—Concentrated Load at Any Point



$$R_1 = \frac{Pb^2(2l+a)}{2l^3}$$

$R_2 = P - R_1$

$V_x = R_1$ (when $x < a$)

$= R_2$ (when $x > a$)

$$M_x = \frac{Pb^2x(2l+a)}{2l^3}$$

(when $x < a$)

$= R_1x - P(x-a)$

(when $x > a$)

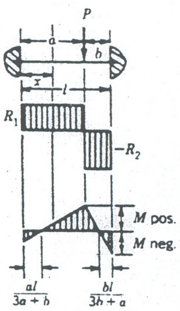
$$M_{positive} = \frac{Pab^2(2l+a)}{2l^3}$$

(when $x = a$)

$$M_{negative} = -\frac{Pab(l+a)}{2l^2}$$

(when $x = l$)

Fixed Beam—Concentrated Load at Any Point



$a > b$

$R_1 = Pb^2(l + 2a)/l^3$

$R_2 = Pa^2(l + 2b)/l^3$

$V_x = R_1$ (when $x < a$)

$= R_2$ (when $x > a$)

$V = R_2$

$$M_x = R_1x - \frac{Pab^2}{l^2}$$

(when $x < a$)

$$= R_2(l-x) - \frac{Pa^2b}{l^2}$$

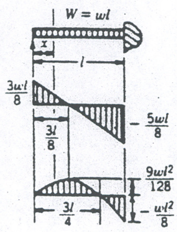
(when $x > a$)

$$M_{positive} = \frac{2Pa^2b^2}{l^3}$$

$$M_{negative} = -\frac{Pa^2b}{l^2}$$

$$y = -\frac{2Pa^3b^2}{3EI(3a+b)^2}$$

Beam Supported at One End, Fixed at Other—Distributed Load



$$R_1 = \frac{3wl}{8}$$

$$R_2 = \frac{5wl}{8}$$

$$V_x = \frac{3wl}{8} - wx$$

$$V = \frac{3wl}{8} \text{ (at left support)}$$

$$= \frac{5wl}{8} \text{ (at right support)}$$

$$M_x = wx\left(\frac{3l}{8} - \frac{x}{2}\right)$$

$$M_{positive} = \frac{9wl^2}{128}$$

$$M_{negative} = -\frac{wl^2}{8}$$

$$y = -\frac{0.0054wl^4}{EI} \text{ (at } 0.4215l \text{ from } R_1)$$