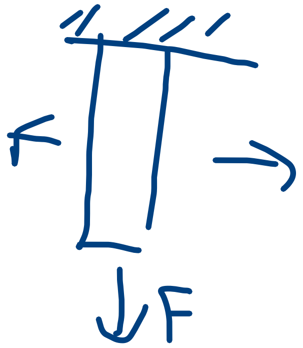
VÍCEOSÁ NAPJATOST

$\underline{\underline{\sigma}}$

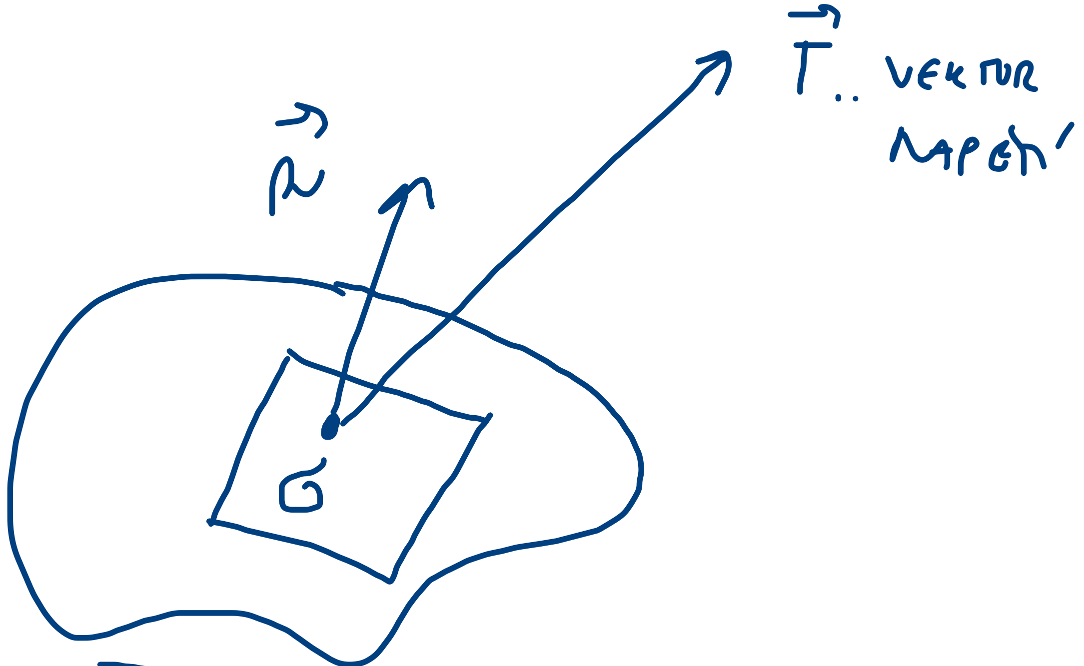
$\underline{\underline{\varepsilon}}$

$\underline{\sigma = E \cdot \varepsilon}$ (JEDNOOSÁ NAPJATOST)



TENZOR NAPĚTÍ

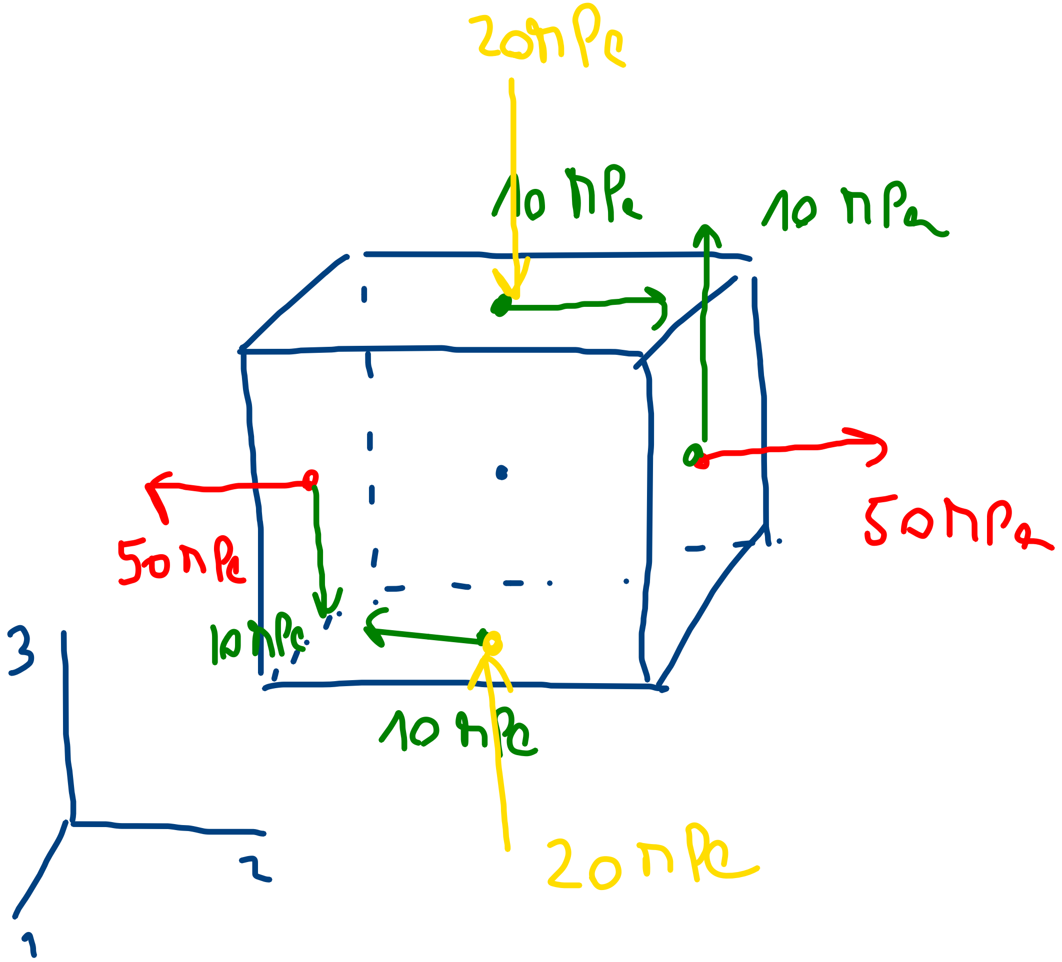
$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$



$$\vec{T} = \underline{\underline{\sigma}} \cdot \vec{n}$$

$$T_i = \sum_{j=1}^{3} \sigma_{ij} \cdot n_j$$

$$T_2 = \sigma_{21} n_1 + \sigma_{22} n_2 + \sigma_{32} n_2$$



$$\sigma_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 50 & 10 \\ 0 & 10 & -20 \end{pmatrix} \text{MPa}$$

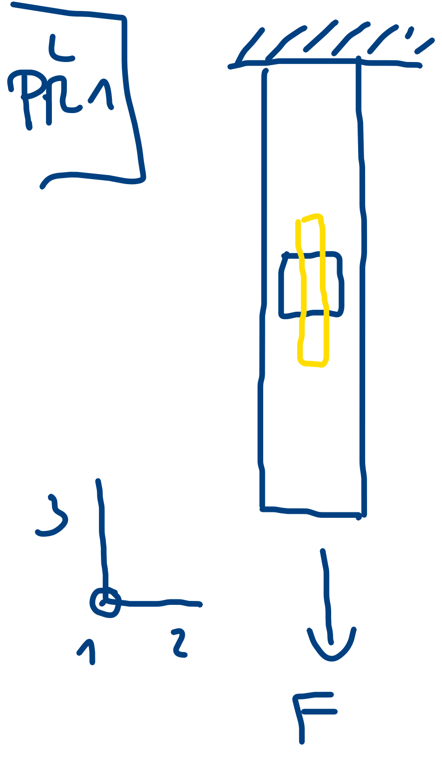
SMYKOVÉ SLOŽKY

NORMÁLOVÉ SLOŽKY

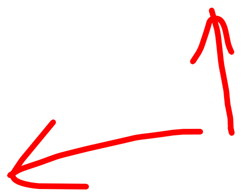
NORMÁLOVÉ SLOŽKY

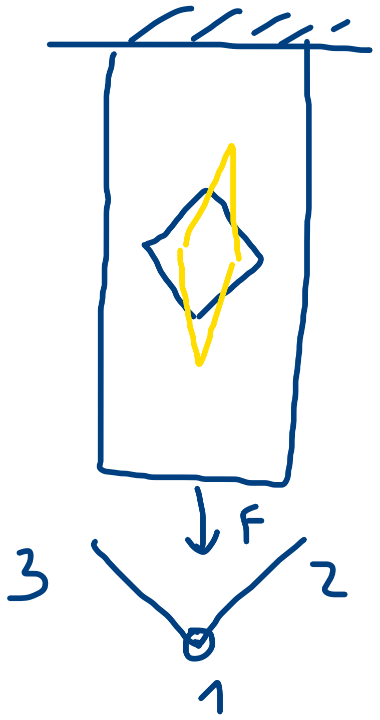
Hlavní směry, hlavní napětí a hlavní deformace

Př 1



$$\sigma_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & F/S \end{pmatrix}$$





$$\sigma_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & F/2S & F/2S \\ 0 & F/2S & F/2S \end{pmatrix}$$

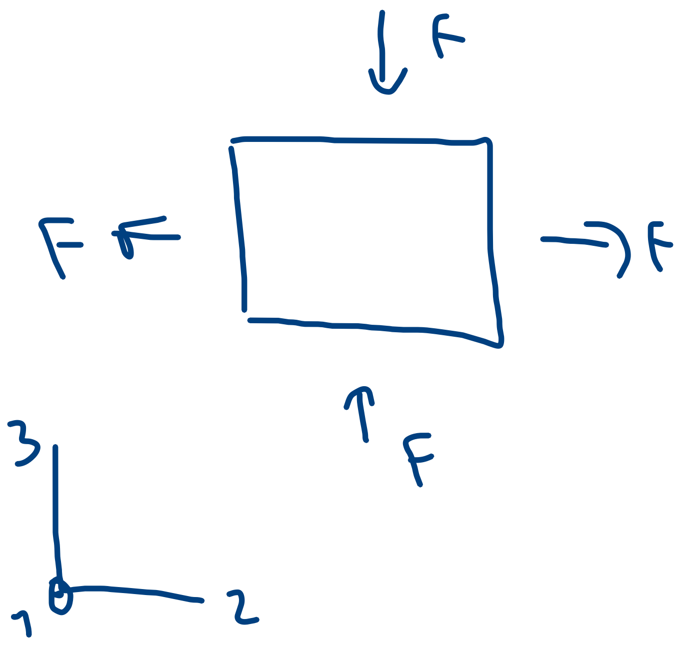
$$\sigma_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

$\sigma_1, \sigma_2, \sigma_3$ .. hlavní napětí

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}$$

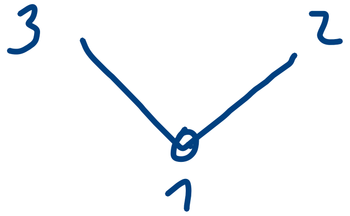
$\varepsilon_1, \varepsilon_2, \varepsilon_3$ .. hlavní deformace

Př 2



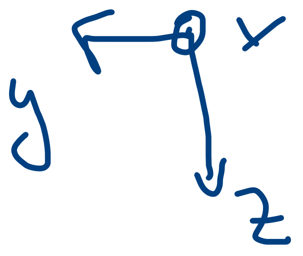
$$\sigma_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & F/S & 0 \\ 0 & 0 & F/S \end{pmatrix}$$

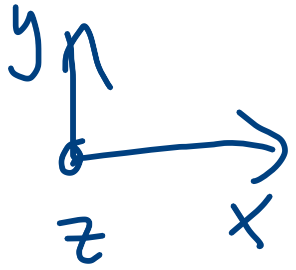
$$\sigma_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & F/S \\ 0 & F/S & 0 \end{pmatrix}$$

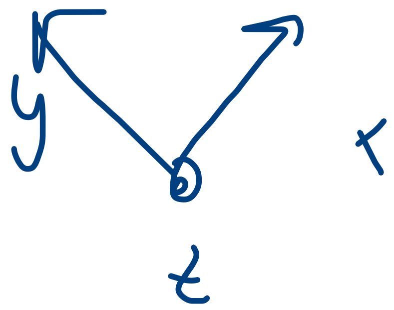


čistý smyk

$$\vec{g} = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} m/s^2 \qquad \vec{g} = \begin{pmatrix} 0 \\ -10 \\ 0 \end{pmatrix} m/s^2 \qquad \vec{g} = \begin{pmatrix} -\frac{10}{\sqrt{2}} \\ -\frac{10}{\sqrt{2}} \\ 0 \end{pmatrix} m/s^2$$







$$\underline{\underline{\sigma}} = \begin{pmatrix} 10 & +20 & 30 \\ +20 & 0 & 50 \\ 30 & 50 & 60 \end{pmatrix} MPa$$

TENZOR NAPĚTÍ
DEFORMACE

<u>SYMETRICKÝ</u>

$\downarrow$

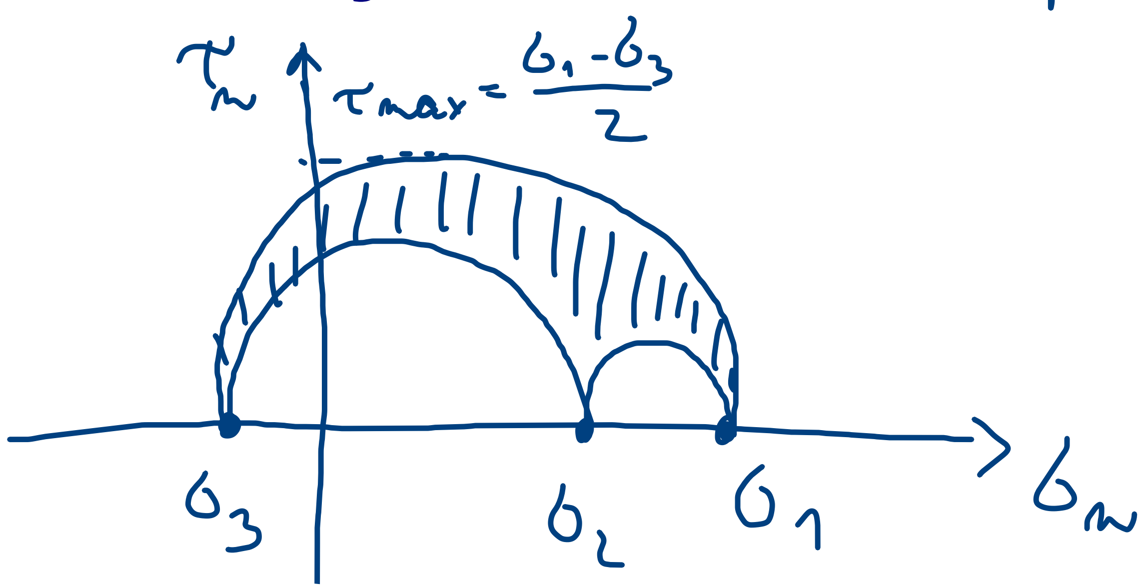
HLAVNÍ NAPĚTÍ ... VLASTNÍ ČÍSLA MATICE

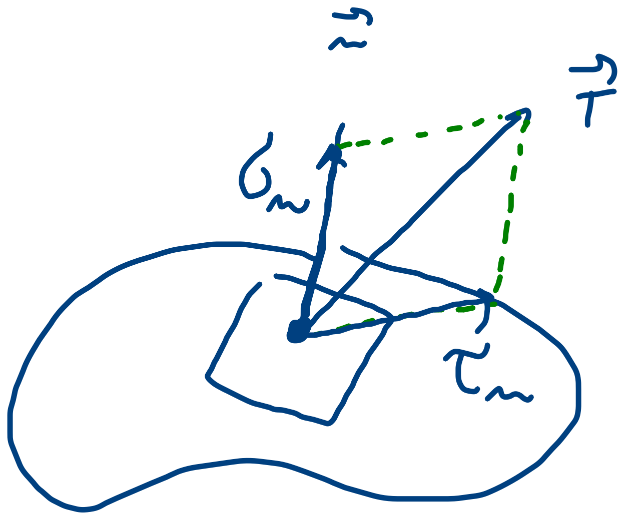
HLAVNÍ SMĚRY ... VLASTNÍ VEKTORY MATICE

Mohrovy kružnice

HLAVNÍ NAPĚTÍ $\sigma_1, \sigma_2, \sigma_3$

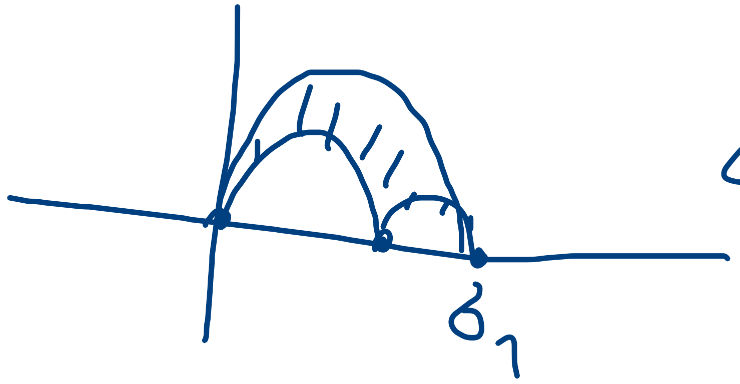
40MPa ↓ 30MPa, ↓ -10MPa



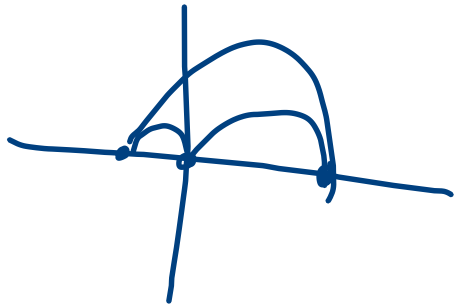


a) 3OSÁ NAPJATOST

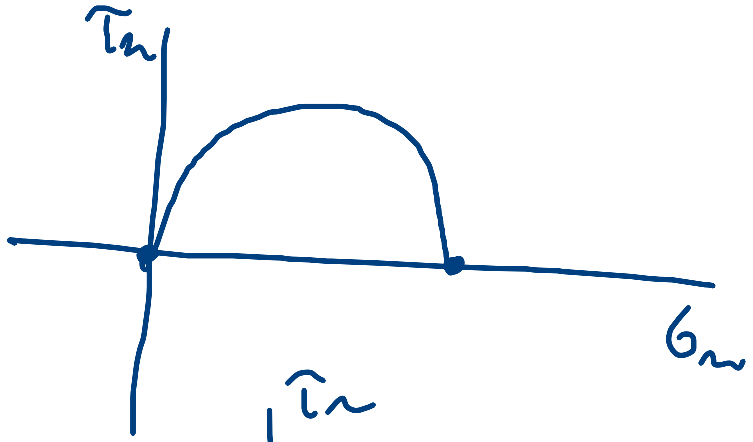
b) DVOUOSÁ NAPJATOST ... $\sigma_3 = 0$



~~$\tau_{max} = \frac{\sigma_1}{2}$~~



c) JEDNOOSÁ NAPJATOST ... $\sigma_2 = \sigma_3 = 0$



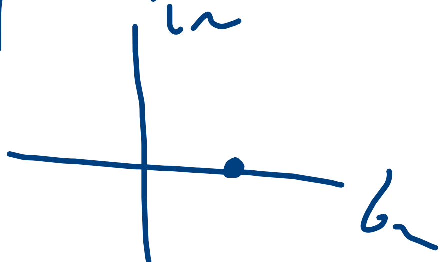
$\tau_{max} = \frac{\sigma_1}{2}$

$\sigma_1 = 2\tau_{max} < \sigma_k$

TRESCA

d) HYDROSTATICKÁ NAPJATOST

$\sigma_1 = \sigma_2 = \sigma_3$



$\tau = 0$

Hookeův zákon při víceosé napjatosti

1osá n. $\sigma = E \cdot \varepsilon$ $\qquad \varepsilon = \frac{1}{E}\sigma$

3osá n. $\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$

~~$\sigma_{ij} = E \cdot \varepsilon_{ij}$~~

~~$\sigma_{23} = E\,\varepsilon_{23}$~~
~~$\sigma_{12} = E\,\varepsilon_{12}$~~

sym $\quad$ ~~81~~ 21 $\quad$ sym

C_{ijkl}
- anizotropní materiál
  - obecný .. 21
  - krychlový .. 3
- izotropní materiál .. 2 (E, μ) → modul ve smyku $G = \frac{E}{2(1+\mu)}$

ortotropní (180°) → 9

Hookeův zákon pro izotropní materiál (3osá napjatost)

$$\sigma_{ij} = \frac{E}{1+\mu}\left[\varepsilon_{ij} + \frac{\mu}{1-2\mu}\,\delta_{ij}\,\varepsilon_{kk}\right]$$

$$\varepsilon_{ij} = \frac{1+\mu}{E}\sigma_{ij} - \frac{\mu}{E}\,\delta_{ij}\,\sigma_{kk}$$

δ_{ij} = Kroneckerova delta $\begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$

$\varepsilon_{kk} = \sum_{k=1}^{3} \varepsilon_{kk}$

$\sigma_{kk} = \sum_{k=1}^{3} \sigma_{kk} = \sigma_{11} + \sigma_{22} + \sigma_{33}$

HOOKEŮV ZÁKON PRO IZOTROPNÍ KONTINUUM

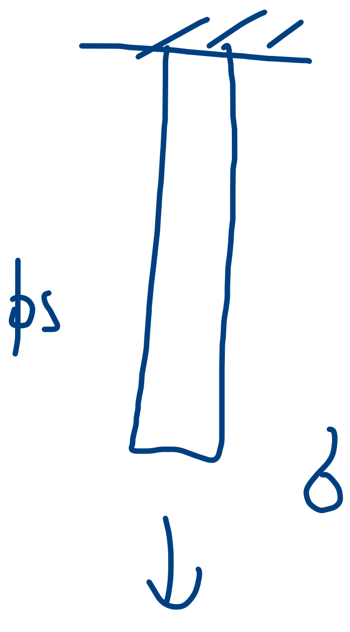
(2OSÁ NAPJATOST)

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \mu\sigma_2)$$
$$\varepsilon_2 = \frac{1}{E}(\sigma_2 - \mu\sigma_1)$$

$$\sigma_1 = \frac{E}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2)$$
$$\sigma_2 = \frac{E}{1-\mu^2}(\varepsilon_2 + \mu\varepsilon_1)$$

~~$\sigma_1 = E\varepsilon_1$~~
~~$\sigma_2 = E\varepsilon_2$~~

PEVNOST

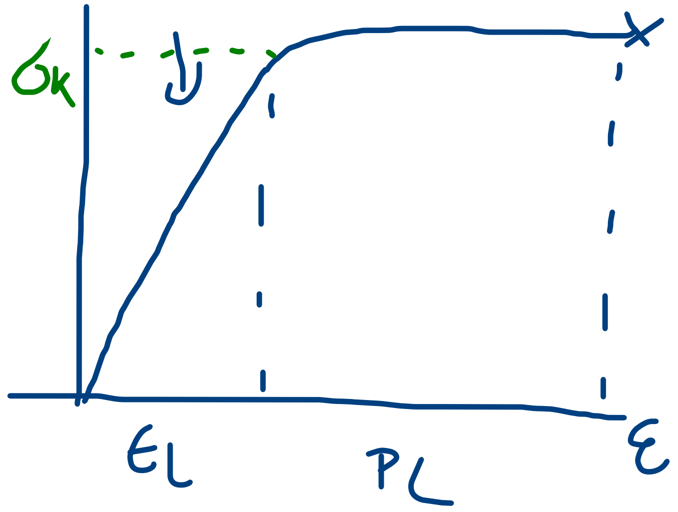


$\sigma = \frac{F}{S}$

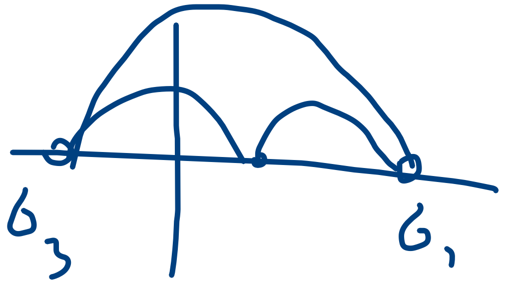
$\sigma_K = 300\ MPa$

$\sigma = E \cdot \varepsilon$

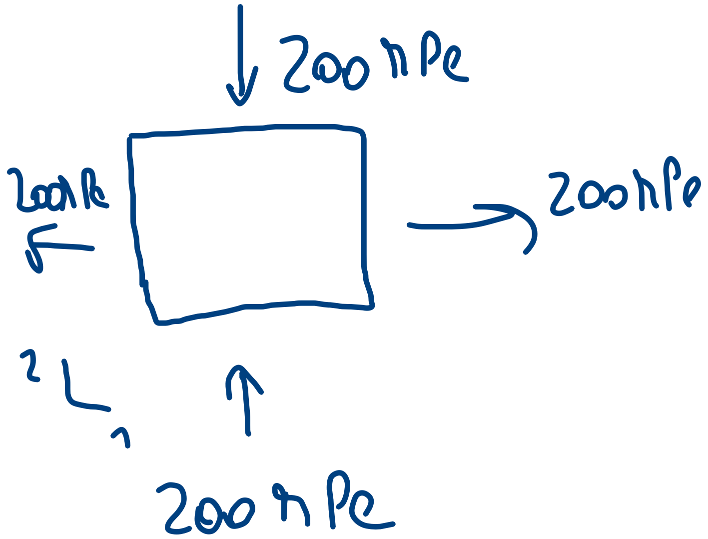
HOOKE



NOSÉ NAMÁHÁNÍ



$$\tau_{max} = \frac{(\sigma_1 - \sigma_3)}{2}$$



$$\sigma_{ij} = \begin{pmatrix} 200 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ldots \quad \sigma_1 = 200\ MPa, \quad \sigma_2 = 200\ MPa, \quad \sigma_3 = 0$$

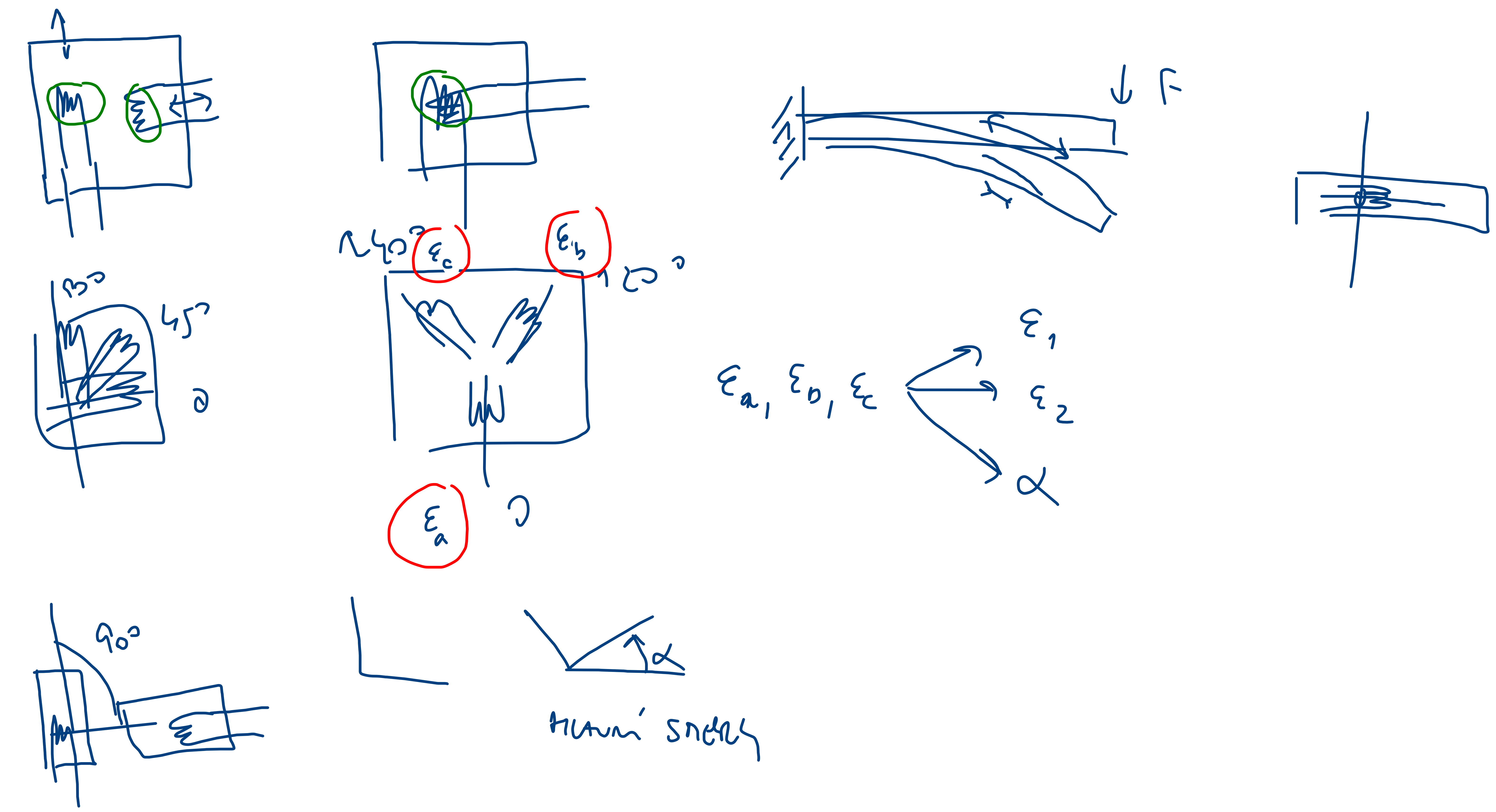
TRESCA .. $\sigma_{RED} = 2 \cdot \tau_{max}$

σ_{RED} { TRESCA / HMH (VON MISES) .. σ_{RED}

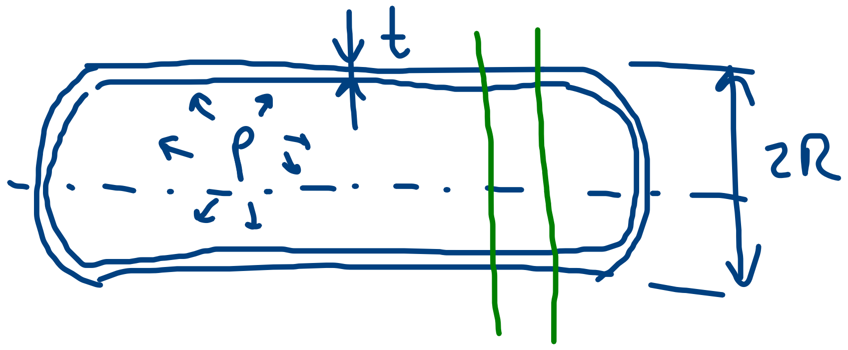
$$\sigma_{RED} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

VON MISESOVO NAPĚTÍ

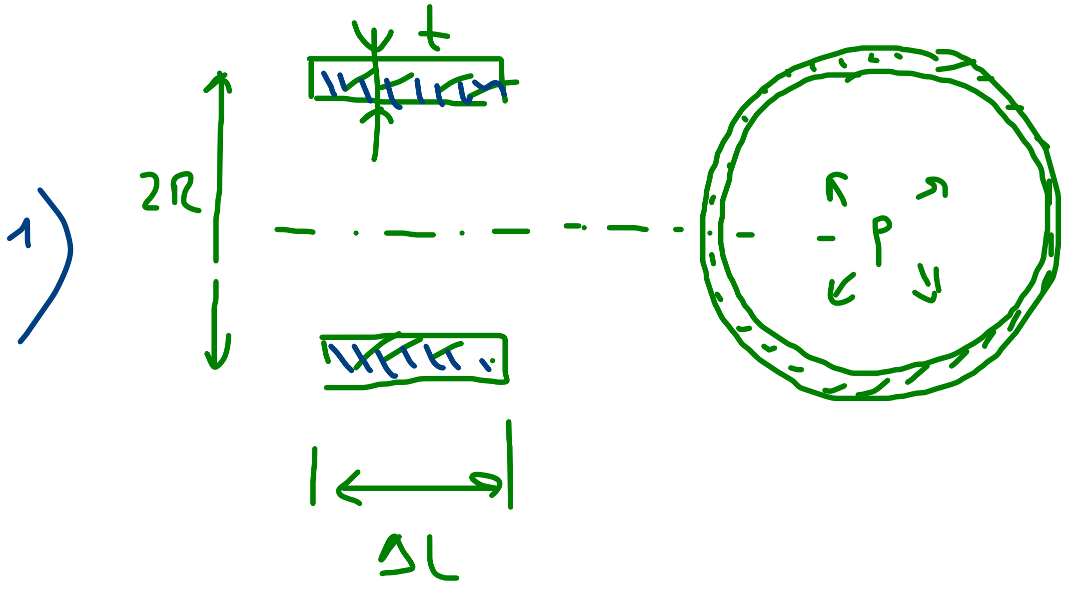
Měření dvouosé napjatosti - tenzometrické růžice (rozety)

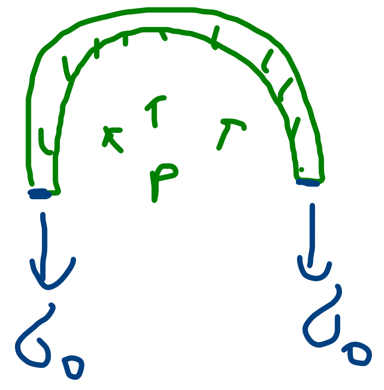


Tenkostěnná válcová nádoba zatížená vnitřním přetlakem



1)



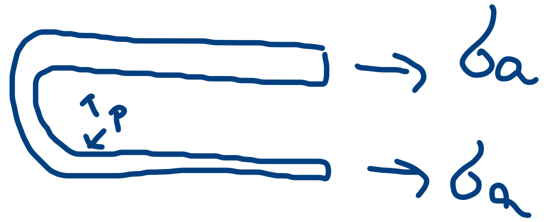


$$\sigma_o \cdot \Delta L \cdot t \cdot 2 = p \cdot 2R \, \Delta L$$

$t \ll R$

$$\boxed{\sigma_o = p \cdot \frac{R}{t}}$$

2) AXIÁLNÍ



$$\sigma_a \cdot 2\pi R \cdot t = p \pi R^2$$

$$\boxed{\sigma_a = p \cdot \frac{R}{2t}}$$

ε_o

ε_a

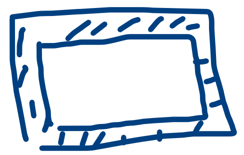
$$\sigma_o = 2 \cdot \sigma_a \qquad \frac{\sigma_o}{\sigma_a} = 2$$

~~$\varepsilon_o = 2\varepsilon_a$~~

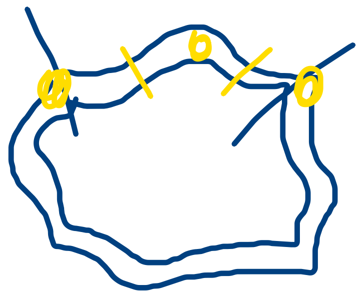
$$\frac{\varepsilon_o}{\varepsilon_a} = ?$$

POZN.: KOULE ○ $\sigma_a = \sigma_o = p \frac{R}{2t}$

POZN 2:



$t, s \; \ldots \; \sigma_{max} \doteq 10 \times \sigma_o$



$\Leftrightarrow$