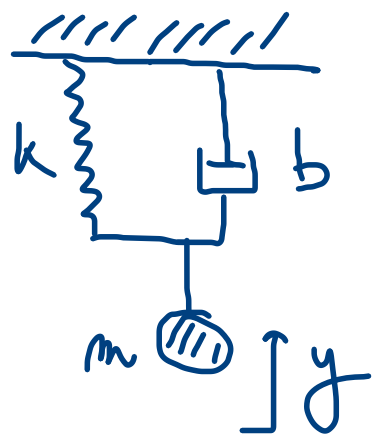


# ΤΛΥΝΕΝΕΊ ΚΗΙΡΑΊΝΊ



$$F_{TL} = b \cdot \dot{y}$$

$$m\ddot{y} + b\dot{y} + ky = 0$$

1) VΛΑΣΤΗΚΗΤΗ

b... ΣΟΥΣΗΜΤΕΛ ΤΛΥΝΕΜΊ

$\delta = \frac{b}{2m}$  ... ΚΟΝΣΤΑΝΤΑ ΎΤΛΩΗ

$$\ddot{y} + \underbrace{\left(\frac{b}{m}\right)}_{2\delta} \dot{y} + \underbrace{\left(\frac{k}{m}\right)}_{\Omega_0^2} y = 0$$

$$\ddot{y} + 2\delta\dot{y} + \Omega_0^2 y = 0$$

... ΠΡΝΕΝΕ  $y(t) = y_0 e^{\alpha t}$

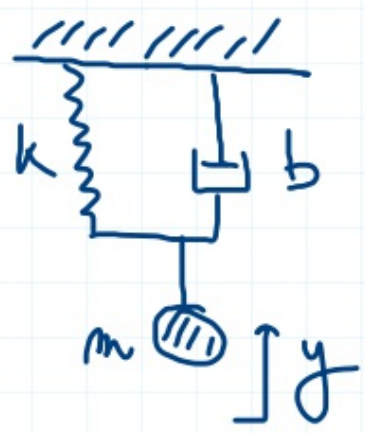
$\alpha \in \mathbb{C}$

$\text{Im}(y_0 e^{\alpha t})$

$$\rightarrow \alpha^2 y_0 e^{\alpha t} + 2\delta\alpha y_0 e^{\alpha t} + \Omega_0^2 y_0 e^{\alpha t} = 0$$

$$\alpha^2 + 2\delta\alpha + \Omega_0^2 = 0$$

$$\rightarrow \alpha_{1,2} = -\delta \pm \sqrt{\delta^2 - \Omega_0^2}$$



$$\alpha_{1,2} = -\delta \pm \sqrt{\delta^2 - \Omega_0^2}$$

$$\delta = \frac{b}{2m}$$

$$D = \frac{\delta}{\Omega_0} \dots \text{POHÉRNHÍ ÚTLUH}$$

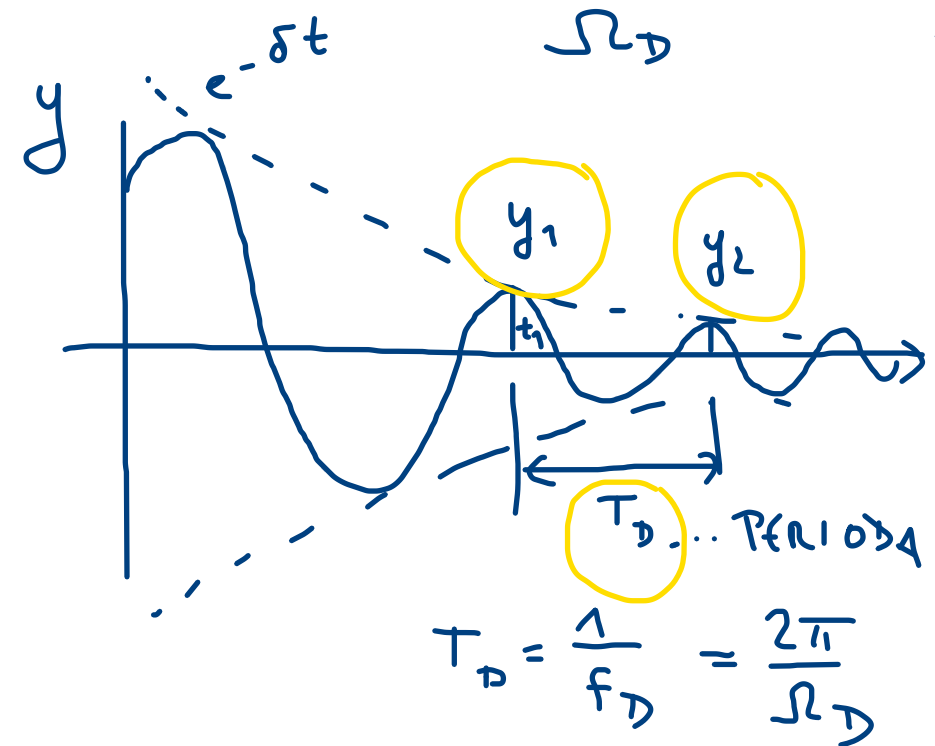
$$y(t) = y_0 e^{\alpha t}$$

1)  $\delta < \Omega_0$  ( $D < 1$ ) ... PODKRITICKÝ ÚTLUH

$\Omega_D$  .. VLASTNÍ FAKT VENCE  
TRUENÍCH KNITŮ

$$\alpha_{1,2} = -\delta \pm i \sqrt{\Omega_0^2 - \delta^2} \dots y(t) = y_0 e^{-\delta t} \sin(\Omega_D t + \psi)$$

$D$  .. LOGARITMICKÝ DEKREMENT  
ÚTLUHU



$$D = \ln \frac{y_1}{y_2} = \ln \frac{y_0 e^{-\delta t_1} \cdot 1}{y_0 e^{-\delta(t_1 + T_D)} \cdot 1} = \ln(e^{-\delta(t_1 - t_1 - T_D)}) =$$

$$= \ln e^{+\delta T_D} = \delta T_D = D$$



2) KRITICKÝ ÚTLUH ..  $\delta = \Omega_0$  ( $D = 1$ )

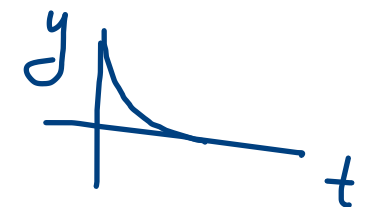
$$y(t) = y_0 e^{-\delta t}$$

3) NADKRITICKÝ ÚTLUH  $\delta > \Omega_0$  ( $D > 1$ )

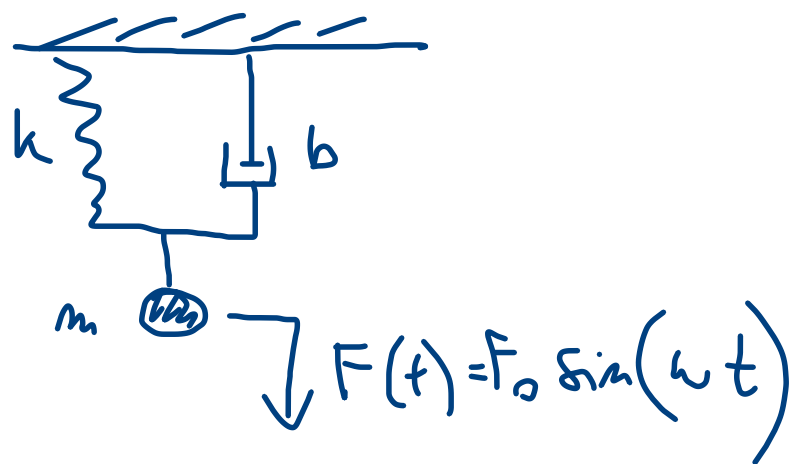
$$\rightarrow \alpha_1 \in \mathbb{R}$$

$$\rightarrow \alpha_2 \in \mathbb{R}$$

$$y(t) = y_{01} e^{-\alpha_1 t} + y_{02} e^{-\alpha_2 t}$$



## 2) VYNUCENÉ KMITY



$$m\ddot{y} + b\dot{y} + ky = F_0 e^{i\omega t}$$

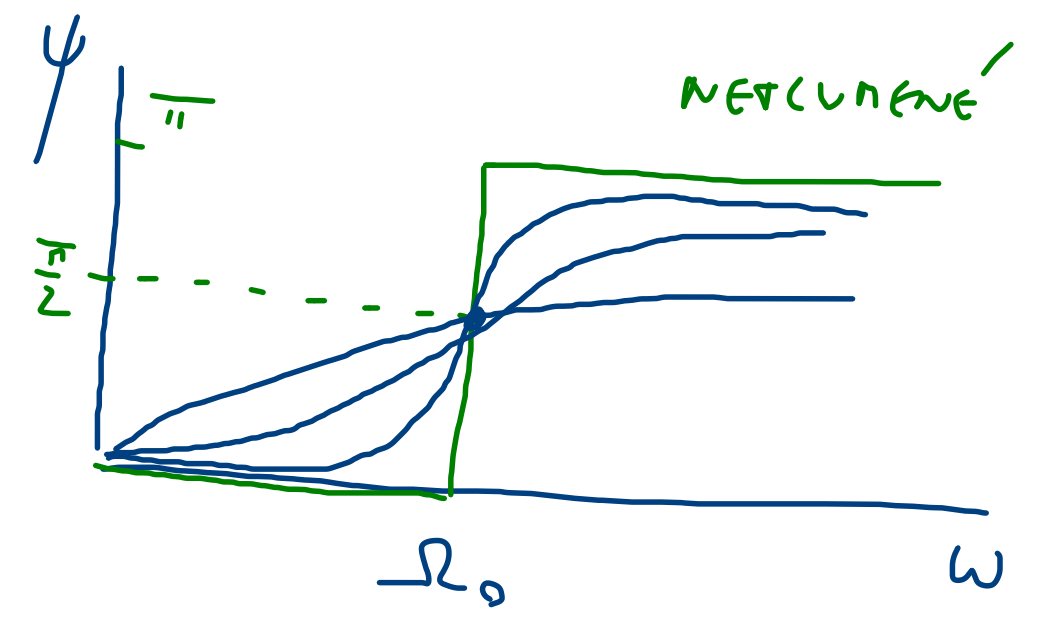
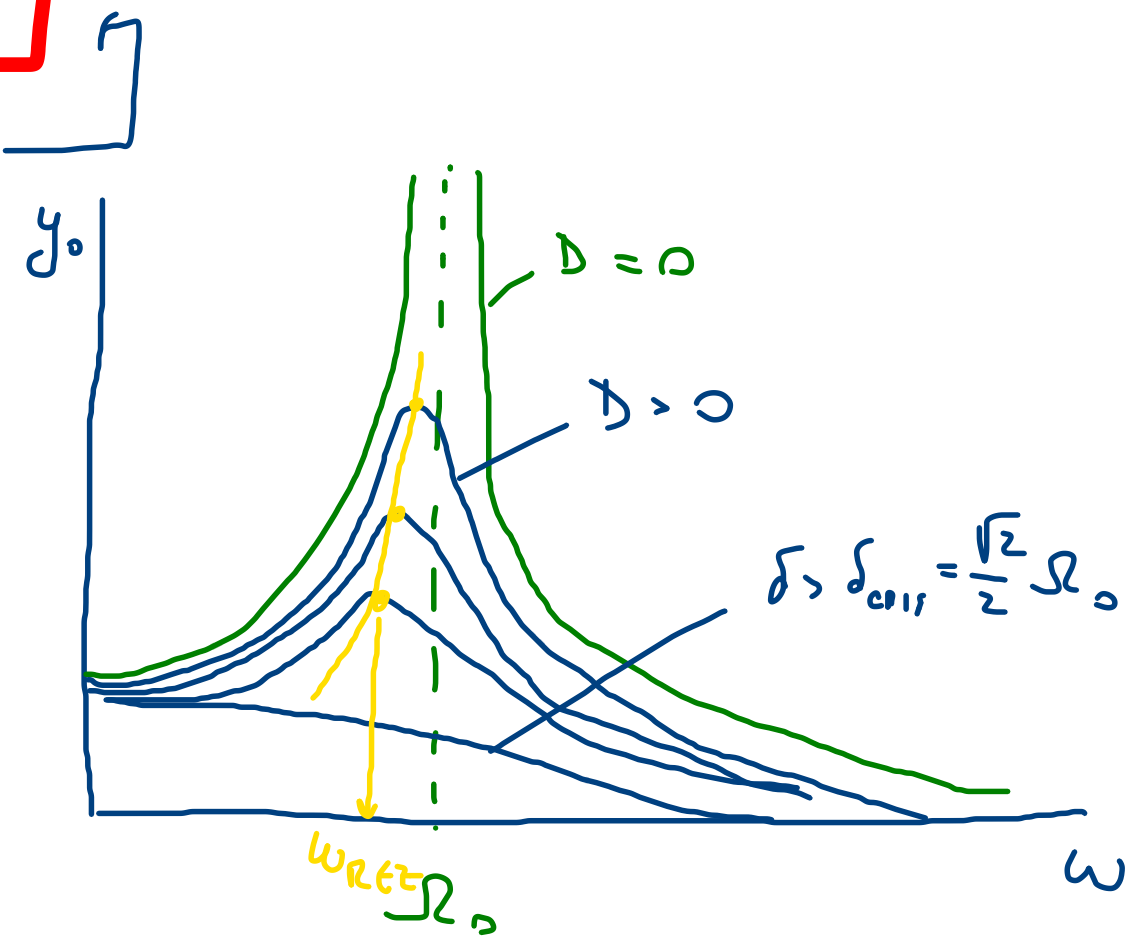
$$y(t) = y_0 e^{i(\omega t + \psi)}$$

$$\rightarrow y_0 = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + b^2\omega^2}} = \frac{F_0/m}{\sqrt{(\Omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2}}$$

$$\frac{d}{d\omega} \left[ (\Omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2 \right] = 0$$

$$2(\Omega_0^2 - \omega^2)(-2\omega) + 4\delta^2 2\omega = 0$$

$$-\Omega_0^2 + \omega^2 + 2\delta^2 = 0 \rightarrow \omega_{REZ} = \sqrt{\Omega_0^2 - 2\delta^2}$$



		NETLUHÉNÉ	TLUHÉNÉ
VLASTNÍ (VOLNÉ)	VLASTNÍ FREQVENCE	$\Omega_0 = \sqrt{\frac{k}{m}}$	$\Omega_D = \sqrt{\Omega_0^2 - \delta^2} = \Omega_0 \cdot \sqrt{1 - D^2}$
VYHNUČNÉ	REZONANČNÍ FREQVENCE	$\omega_{REZ} = \Omega_0$	$\omega_{REZ} = \sqrt{\Omega_0^2 - 2\delta^2}$