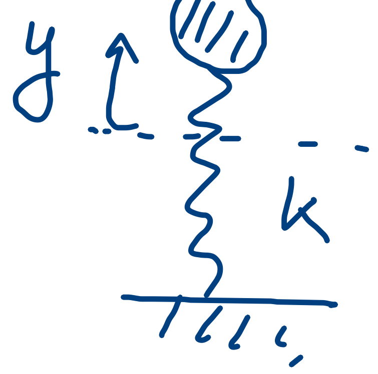
TEORIE KMITÁNÍ

- VOLNÉ – ZAJÍMÁ NÁS FREKVENCE Ω
- VYNUCENÉ – VÍME FREKVENCI ω, ZAJÍMÁ NÁS AMPLITUDA



POČET STUPŇŮ VOLNOSTI (PSV)

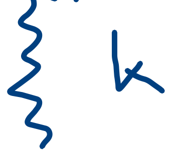
1) HMOTA (SOUSTŘEDĚNÁ)

m — NOSITELKA KIN. ENERGIE

$T = \frac{1}{2} m v^2$

$F_s = m \ddot{y}$

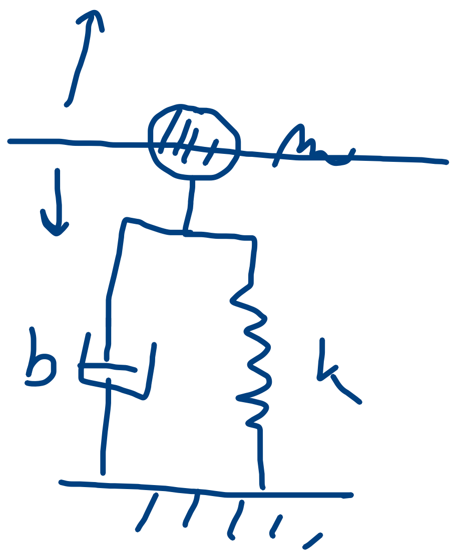
2) TUHOST (PRUŽINA)



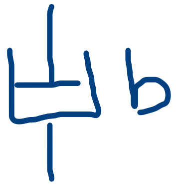
NOSITELKA POT. ENERGIE

$U = \frac{1}{2} k y^2$

$F_{el} = k \cdot y$



3) LINEÁRNÍ TLUMIČ



DISIPUJE ENERGII

$F_t = b \cdot \dot{y}$

NETLUMENÉ KMITÁNÍ SOUSTAV SE SOUSTŘEDĚNÝMI PARAMETRY (TUHÝCH TĚLES)

1) NÁHRADNÍ SYSTÉM

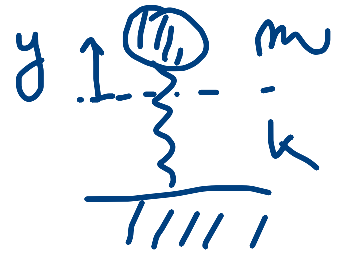
2) SESTAVENÍ POHYBOVÝCH ROVNIC
   - UVOLŇOVACÍ METODA
   - LAGRANGEOVY ROVNICE

3) ŘEŠENÍ POH. RC.

LAGRANGEŮV FORMALISMUS

1. q_i .. ZOBECNĚNÉ SOUŘADNICE $\quad i = 1..PSV$
2. $L = T - U \qquad L(q_i, \dot{q}_i)$
3. LAGR. ROVNICE 2. DRUHU $\quad \boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0} \quad j = 1..PSV$

1) systém s 1 SV



$F_{el} = -k \cdot y$

$F_s = -m\ddot{y}$

$+ky + m\ddot{y} = 0$

$$\boxed{\ddot{y} + \frac{k}{m} y = 0} \quad y(t) \quad \ldots \quad \text{PP}: \; \dot{y}(t=0) = v_0, \quad y(t=0) = y_0$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0}$$

$q_i \; \ldots \; i = 1 \ldots \text{PSV}$

$q_1 = y$

$L = L(q_1, \dot{q}_1) =$
$= L(y, \dot{y})$

$$L = T - U = \frac{1}{2} m v^2 - \frac{1}{2} k \cdot y^2 =$$
$$= \frac{1}{2} m \dot{y}^2 - \frac{1}{2} k y^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = 0$$

$$\frac{d}{dt}\left(\frac{1}{2} m 2\dot{y} - 0\right) - \left(0 - \frac{1}{2} k 2y\right) = 0$$

$$\boxed{m\ddot{y} + ky = 0}$$

$$\boxed{\ddot{y} + \frac{k}{m} y = 0}$$

a) VOLNÉ KMITY

$$y(t) = y_0 \cdot \sin(\Omega t)$$

$$\rightarrow \quad y_0(-\Omega^2)\cancel{\sin(\Omega t)} + \frac{k}{m}\cancel{y_0 \sin(\Omega t)} = 0$$

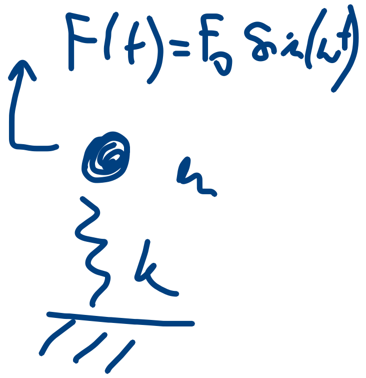
$$\Omega^2 = \frac{k}{m} \quad \rightarrow \quad \boxed{\Omega = \sqrt{\frac{k}{m}}}$$

... KRUHOVÁ FREKVENCE VLASTNÍCH (VOLNÝCH KMITŮ) $\Omega = 2\pi f$

b) VYNUCENÉ KMITY

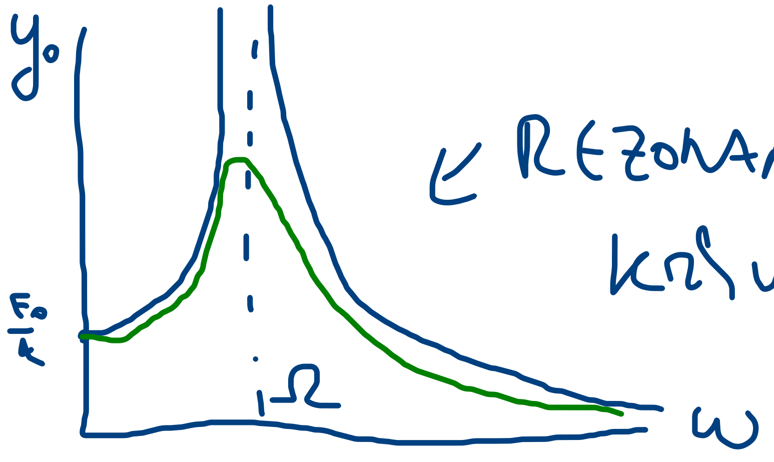
$$\boxed{\ddot{y} + \frac{k}{m} y = F_0 \sin(\omega t)}$$

$$y(t) = y_0 \sin(\omega t - \varphi)$$

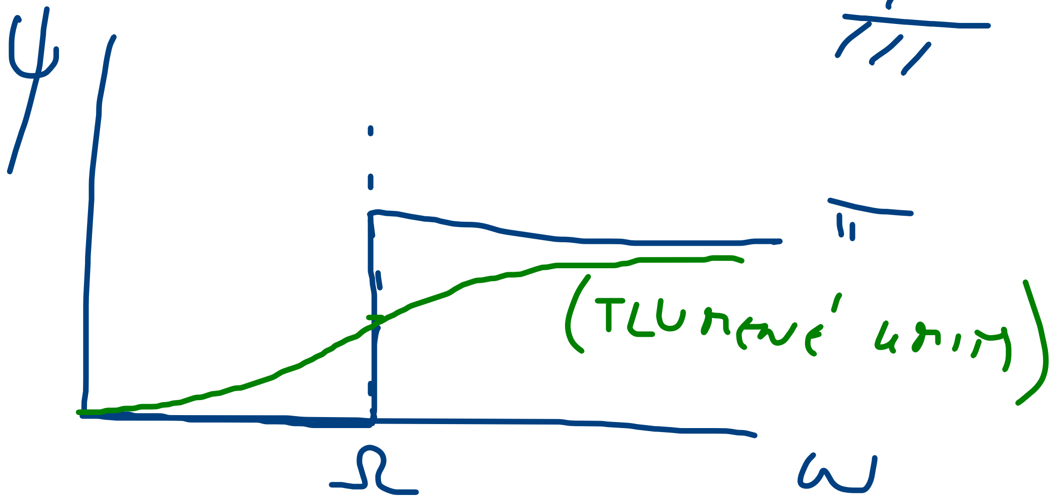


$$y_0 = \frac{F_0}{k - m\omega^2}$$

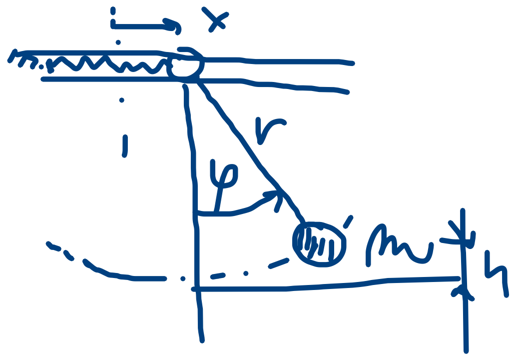
$$\omega \rightarrow \sqrt{\frac{k}{m}} = \Omega$$



← REZONANČNÍ KŘIVKA



2) SYSTÉM SE 2 SV



$q_1 \equiv x$

$q_2 \equiv \varphi$

$L = L(x, \varphi, \dot{x}, \dot{\varphi})$

MALÉ VÝCHYLKY

$\cos\varphi = (1-$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0}$$

$$v_x = (x + r\sin\varphi)^{\cdot} \approx (x + r\varphi)^{\cdot} = \dot{x} + r\dot{\varphi}$$

$$U = \frac{1}{2}kx^2 + mgh = \frac{1}{2}kx^2 + mgr(1-\cos\varphi) \approx \frac{1}{2}kx^2 + mgr\frac{\varphi^2}{2}$$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(v_x^2 + v_y^2) \approx \frac{1}{2}mv_x^2 = \frac{1}{2}m(\dot{x} + r\dot{\varphi})^2$$

$$L = T - U = \frac{1}{2}kx^2 + mgr\frac{\varphi^2}{2} - \frac{1}{2}m(\dot{x} + r\dot{\varphi})^2$$

$i = 1 \ldots q_1 = x$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$

$$\frac{d}{dt}\left(+\cancel{\tfrac{1}{2}}m\,\cancel{2}(\dot{x} + r\dot{\varphi})\right) + \cancel{\tfrac{1}{2}}k\,\cancel{2}x = 0$$

$$\boxed{m(\ddot{x} + r\ddot{\varphi}) + kx = 0} \qquad x(t) \quad \varphi(t)$$

$i = 2 \ldots q_2 = \varphi$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = 0$

$$\frac{d}{dt}\left(+\cancel{\tfrac{1}{2}}m\,\cancel{2}(\dot{x} + r\dot{\varphi})\cancel{r}\right) + \cancel{m}g\cancel{r}\,\cancel{\tfrac{1}{2}}\,\cancel{2}\varphi = 0$$

$$1\,\boxed{\ddot{x} + r\ddot{\varphi} + g\varphi = 0} \qquad x(t) \quad \varphi(t)$$

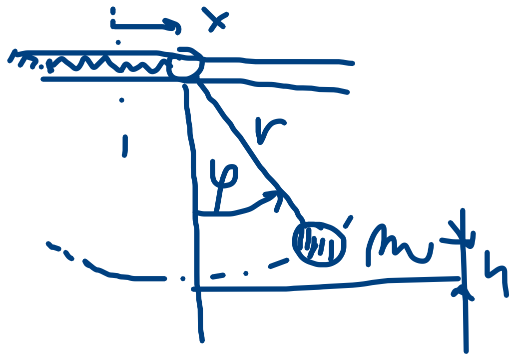
$$\mathbb{M} = \begin{pmatrix} m & mr \\ 1 & r \end{pmatrix} \qquad \mathbb{K} = \begin{pmatrix} k & 0 \\ 0 & g \end{pmatrix}$$

MATICE HMOTNOSTI   MATICE TUHOSTI

$$w(t) = \begin{bmatrix} x(t) \\ \varphi(t) \end{bmatrix}$$

$$\boxed{\mathbb{M}\ddot{w}(t) + \mathbb{K}w(t) = 0}$$

2) SYSTÉM SE 2 SV



$q_1 \equiv x$

$q_2 \equiv \varphi$

$$L = L(x, \varphi, \dot{x}, \dot{\varphi})$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0}$$

$$x(t) = x_{01}\sin(\Omega_1 t) + x_{02}\sin(\Omega_2 t)$$

$$\varphi(t) = \varphi_{01}\sin(\Omega_1 t) + \varphi_{02}(\sin(\Omega_2 t)$$

$$U = \frac{1}{2}kx^2 + mgh - \frac{1}{2}kx^2 + mgr(1-\cos\varphi) \approx \frac{1}{2}kx^2 + mgr\frac{\varphi^2}{2}$$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(v_x^2 + v_y^2) \approx \frac{1}{2}mv_x^2 = \frac{1}{2}m(\dot{x} + r\dot{\varphi})^2$$

$$L = T - U = \frac{1}{2}kx^2 + mgr\frac{\varphi^2}{2} - \frac{1}{2}m(\dot{x} + r\dot{\varphi})^2$$

$i = 1 \,..\, q_1 = x$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$

$$\frac{d}{dt}\left(+\frac{1}{\cancel{2}}m\,\cancel{2}(\dot{x} + r\dot{\varphi})\right) + \frac{1}{\cancel{2}}k\,\cancel{2}x = 0$$

$$\boxed{m(\ddot{x} + r\ddot{\varphi}) + kx = 0} \quad \begin{matrix} x(t) \\ \varphi(t) \end{matrix}$$

$i = 2 \,..\, q_2 = \varphi$: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = 0$

$$\frac{d}{dt}\left(+\frac{1}{\cancel{2}}\cancel{m}\,\cancel{2}(\dot{x} + r\dot{\varphi})\cancel{r}\right) + \cancel{m}g\cancel{r}\frac{1}{\cancel{2}}\cancel{2}\varphi = 0$$

$$\boxed{1\,\ddot{x} + r\ddot{\varphi} + g\varphi = 0} \quad \begin{matrix} x(t) \\ \varphi(t) \end{matrix}$$

$$\mathbb{M} = \begin{pmatrix} m & mr \\ 1 & r \end{pmatrix} \qquad \mathbb{K} = \begin{pmatrix} k & 0 \\ 0 & g \end{pmatrix}$$

MATICE HMOTNOSTI $\qquad$ MATICE TUHOSTI

$$w(t) = \begin{bmatrix} x(t) \\ \varphi(t) \end{bmatrix} \qquad \boxed{\mathbb{M}\,\ddot{w}(t) + \mathbb{K}\,w(t) = 0}$$

$$w(t) = w_0 e^{i\Omega t} \qquad w_0 = \begin{bmatrix} x_0 \\ \varphi_0 \end{bmatrix}$$

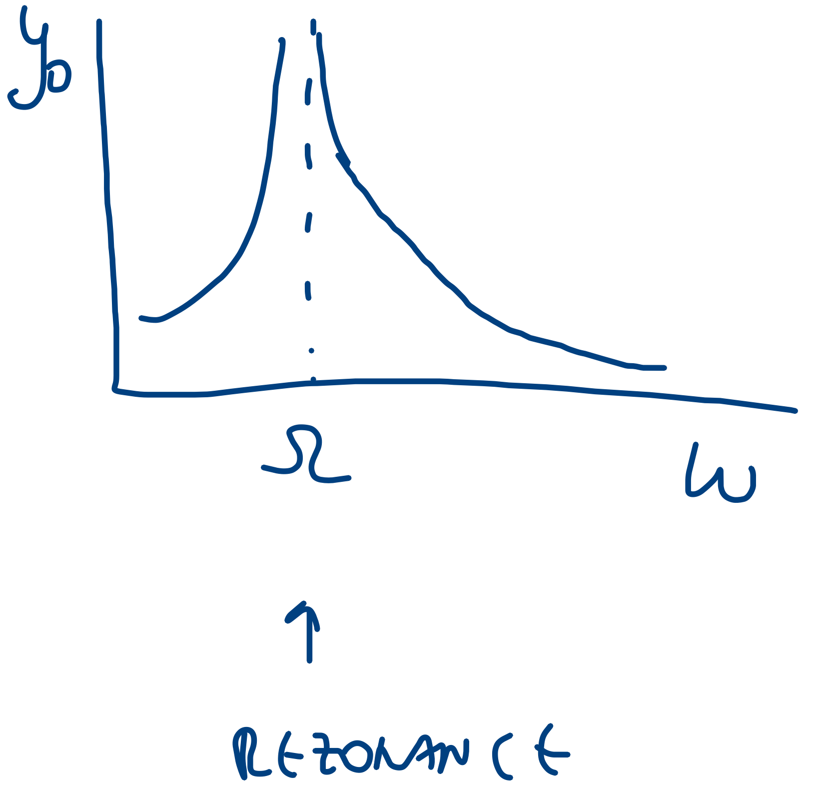
$$(-\Omega^2)\mathbb{M}\,w_0 \cancel{e^{i\Omega t}} + \mathbb{K}\,w_0 \cancel{e^{i\Omega t}} = 0 \quad \ldots -\Omega^2\mathbb{M}\,w_0 + \mathbb{K}\,w_0 = 0$$

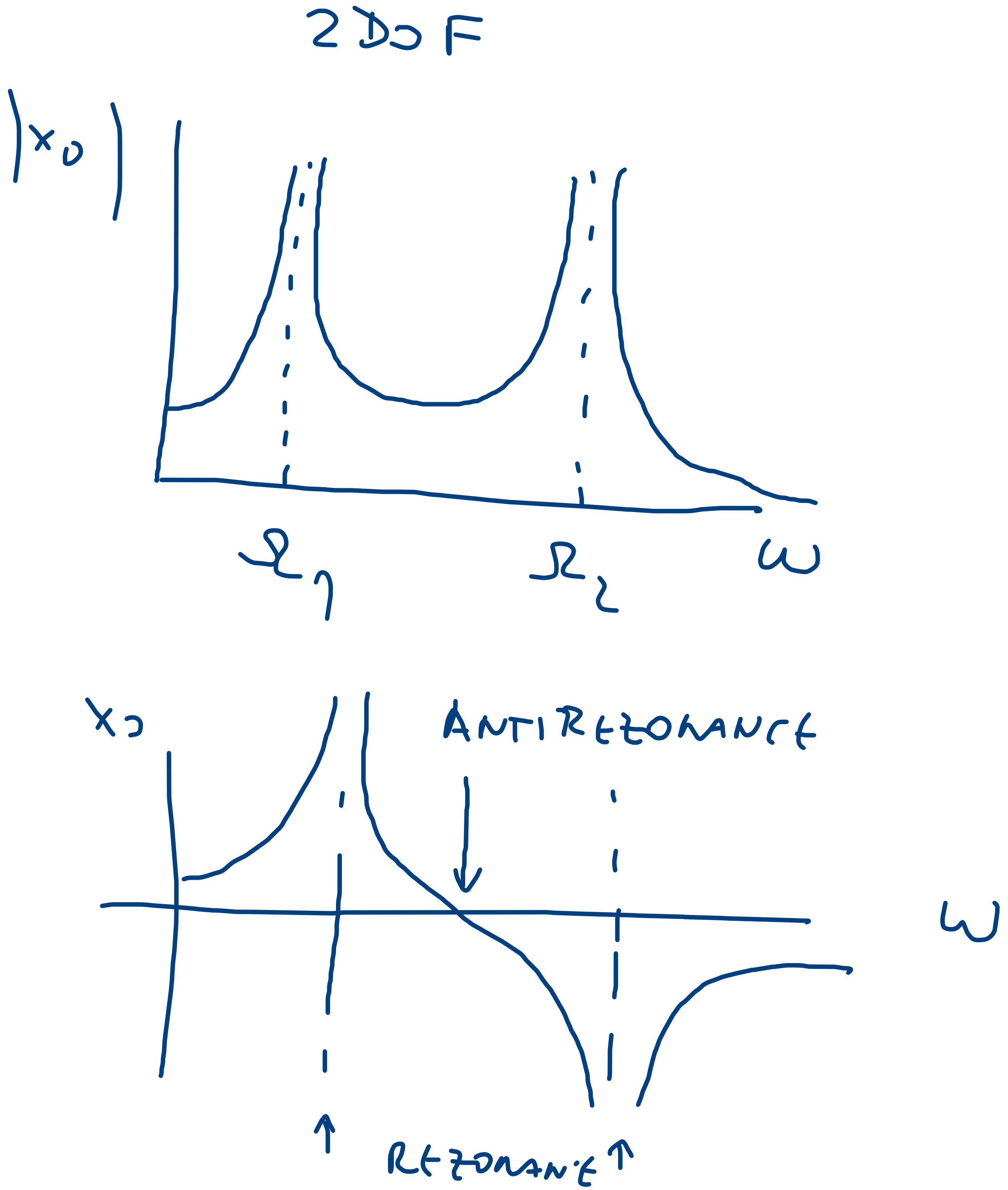
$$(-\Omega^2\mathbb{M} + \mathbb{K})\,w_0 = 0 \iff \det(-\Omega^2\mathbb{M} + \mathbb{K}) = 0$$

$$\downarrow \quad \Omega_1, \Omega_2$$

KRUH. VL. FREKVENCE

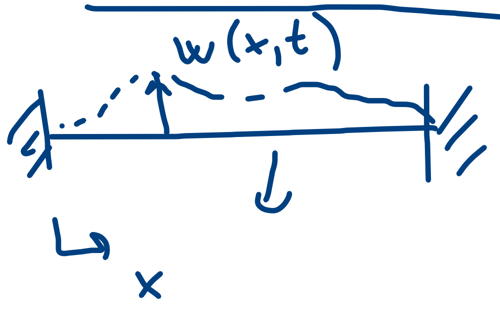
b) VYNUCENÉ KMITY





KMITÁNÍ ELASTICKÝCH TĚLES

- STRUNA



$$\boxed{\frac{\mu}{T}\frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial x^2}}$$

$\mu = \frac{m}{L}$

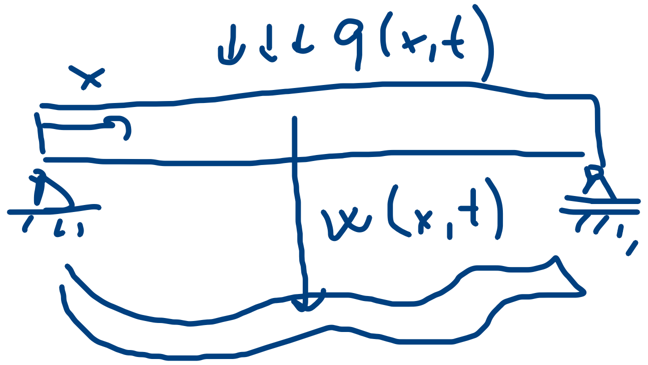
T.. NAPNUTÍ STRUNY

$w(x,t) = w(x)\,\underbrace{T(t)}_{\sin(\Omega_n t)}$

$\Omega_n = 2\pi f_n$

$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$ $\sim n$

- PODÉLNÉ KMITY TYČE
- TORZNÍ KMITY TYČE
- PŘÍČNÉ KMITY NOSNÍKŮ



$$\boxed{A\rho\frac{\partial^2 w}{\partial t^2} + EJ\frac{\partial^4 w}{\partial x^4} = q(x,t)}$$

A [m²].. PRŮŘEZ

ρ ... HUSTOTA

E.. YOUNGŮV MODUL

$J = \frac{1}{12} b h^3$

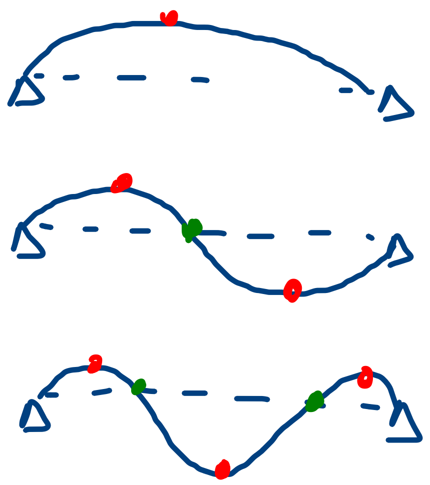
KMITNA

UZEL

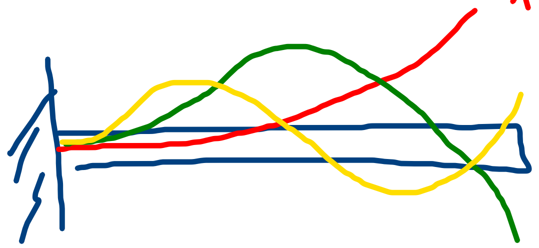
→ 1. MÓD Ω_1

2. MÓD Ω_2

3. MÓD Ω_3



$$\Omega_n = \sqrt{\frac{EJ}{\rho A}}\left(\frac{n\pi}{L}\right)^2 \sim n^2$$

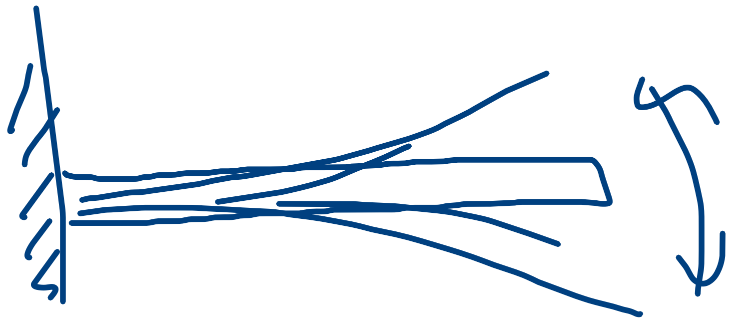


$$\Omega_n = k_n^2\sqrt{\frac{EJ}{\rho A}}$$

$k_1 = \frac{1.875}{L}$

$k_2 = \frac{4.69}{L}$

$k_3 \dots$



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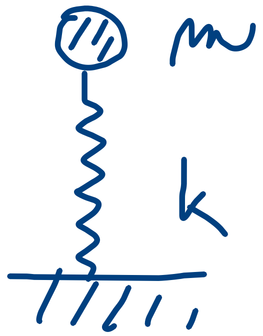
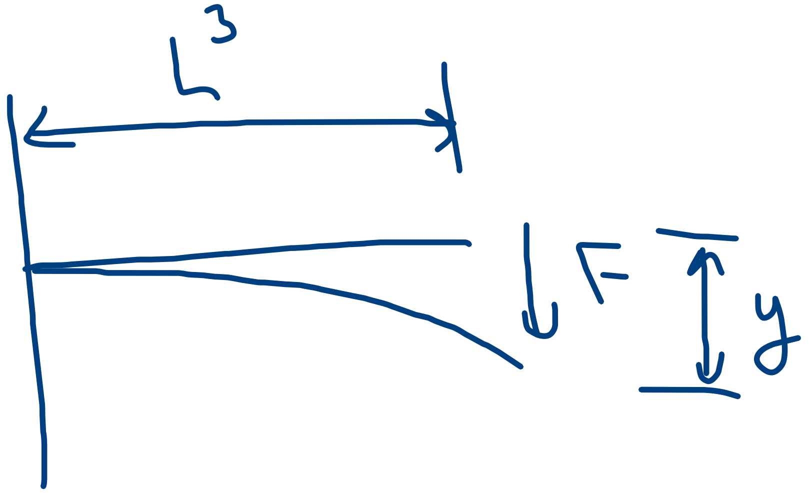


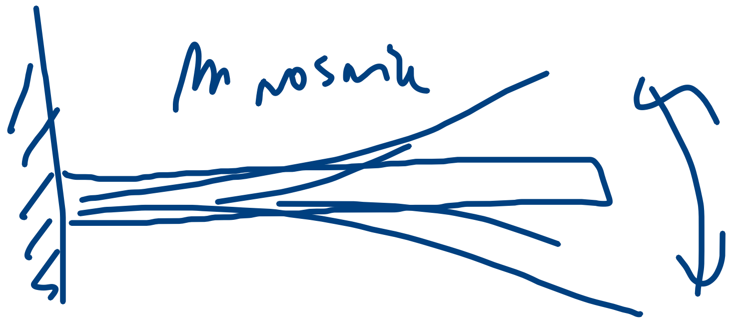
m_{RED} .. REDUKOVANÁ HMOTNOST



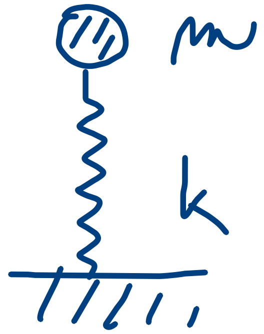
$$F = k \cdot y$$

$$k = \frac{3EJ}{L^3}$$

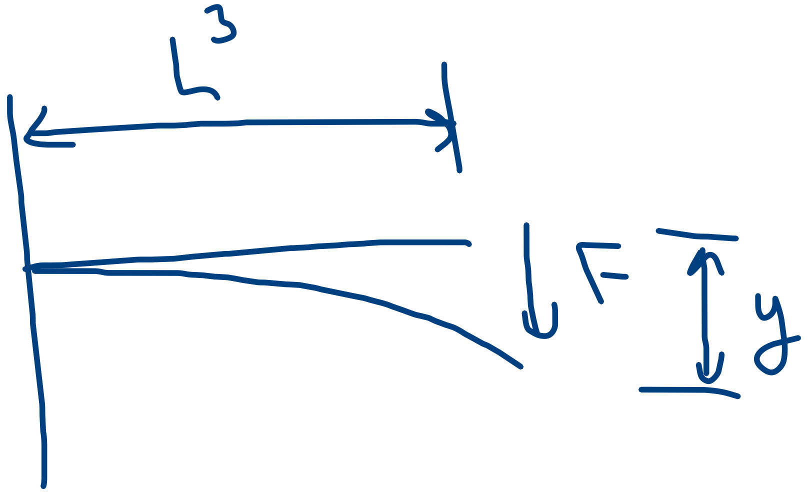
$$J_{\square} = \frac{1}{12} b h^3$$



$\Omega_1 \sim = \Omega$



m_{RED} .. REDUKOVANÁ HMOTNOST



$m_{RED} \doteq 0.24\, m_{nosník}$

$$\Omega = \sqrt{\frac{k}{m_{RED}}}$$

$$F = k \cdot y$$

$$k = \frac{3EJ}{L^3}$$

$$J_{\square} = \frac{1}{12} b h^3$$